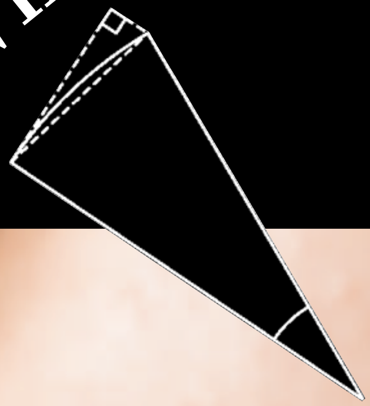
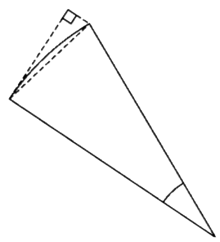
At the transition to university

Newton's Mechanics: Who Needs It?



Contents

Newton's Mechanics: Who Needs It?



REPORT
on the mechanics problem at the transition to university

Newtonian mechanics plays a vital role in preparing students at 16-19 for physics, engineering and applied mathematics at university but each year more students arrive with little or no experience of it.

There has been a significant decline in the take-up of mechanics following curriculum changes in 2004 and students in 30-40% of schools and colleges now have minimal access to mechanics modules (at most one).

UK engineering, science and industry prize highly the skills of mathematical modelling and problem solving grounded in Newton's mechanics.

Urgent action is needed to increase access to and uptake of Newton's mechanics at ages 16-19.

findings and recommendations of a two day symposium
at the Møller Centre, Cambridge, 30th June, 1st July 2008

Mike Savage and Charlie Stripp

Preface

For over fifty years the study of Newtonian mechanics in schools and colleges has played a crucial role in preparing students for physics, engineering and applied mathematics at university. By applying Newton's laws of motion to a wide range of physical problems students are able to develop the vital mathematical modelling and problem solving skills needed to apply mathematics effectively.

As well as their traditional application in science and engineering, these transferable skills are highly prized and sought after in today's economy since, once acquired, they open the door to problem solving in many other fields. Indeed the 2008 crisis within the financial sector clearly illustrates an urgent need for the formulation of robust mathematical models that will underpin and ensure stability of financial systems and the wider economy.

Recent evidence from schools, colleges and universities, however, reveals a major problem at the transition to university. In a substantial number of schools and colleges students have, for various reasons, limited access to mechanics modules as part of A level Mathematics; many students are now only able to study one module at most. As a result there is a widening gap between students' experience of Newtonian mechanics and the expectations of many science and engineering departments of the knowledge and problem-solving skills that students will bring with them. This has created a mechanics problem at the transition to university.

This report is the outcome of a two-day symposium, chaired by Sir Peter Williams and bringing together the interests of schools, colleges, universities and those concerned with UK science from industry and business.

Evidence was shared of the extent and implications of the problem but there was also encouraging news of successful developments to address it and on which to build. This includes the work of the Further Mathematics Network in increasing student access to mechanics modules, teaching and resources. It includes initiatives in the Universities of Manchester, Bristol and others where links are being strengthened with mathematics and science departments in their local schools and colleges. The findings and recommendations which have emerged from the symposium indicate priorities for action and propose practical measures for achieving change.

This report is aimed at those in universities with responsibility for admissions in mathematics and science-based subjects, particularly physics and engineering, and wherever mathematical modelling and problem-solving are key skills. It is further aimed at those in schools with a responsibility for giving students access to studying mechanics and further mathematics. It aims to influence those involved with the design and content of mathematics, physics and engineering curricula at this level, and all those who wish to inspire young people with the excitement, skills and opportunities which Newton's mechanics opens up for them.

The mechanics problem at the transition to university

Almost ten years ago a symposium was held at the Møller Centre on what became known as the mathematics problem at the school/university interface. Evidence was presented of a rapid decline in students' pure mathematics skills on entry to university and the report of that symposium, Measuring the Mathematics Problem (2000), made an immediate impact. The action taken on its recommendations has played an important part in increasing the confidence of many students with mathematics and easing their transition to university.

Unfortunately an applied mathematics problem has now become apparent in Higher Education. Evidence shows that while first year undergraduates in mathematics, physics and engineering are now generally more proficient in pure mathematics, many of them cannot apply it!

Between 2001 and 2003, staff in the School of Physics and Astronomy at the University of Leeds observed that a significant number of first year students were unable to apply their mathematics to the solution of physical problems in the core subjects of mechanics and electromagnetism. They also observed that approximately 10% of the cohort had studied no mechanics modules as part of A level mathematics, 30% had taken only one module, leaving only 60% with the two or more modules that had been expected. Was there a connection? Analysis showed a direct link between students' ability to solve physical problems and the number of modules of mechanics studied.

The concept of a mechanics problem at the transition to university prompted wider investigation and studies were initiated in 2003 by Stephen Lee from Loughborough University, by Dick Clements in Bristol, and most recently by Gareth Jones across European universities, including the UK. Research from the Universities of Bristol and Loughborough showed that fewer students starting engineering degree courses had taken two or more modules of mechanics.

In 2004 the A level Mathematics curriculum was restructured by reducing the applied mathematics content from three modules to two, allowing more time for students to master the pure mathematics. As expected, this has led to a welcome improvement in students' grounding in pure mathematics and numbers taking A level Mathematics are increasing. Predictably, it has also resulted in fewer students taking more than one mechanics module.

Meanwhile, Sue de Pomerai has used data from the awarding bodies to research the impact of the 2004 curriculum change in schools and colleges. This confirms a significant decline in the take-up of mechanics modules. A key finding is that students in 30-40% of schools and colleges have very limited access to mechanics - only one module at most!

With mounting evidence of the impact of curriculum change in schools and colleges, fewer students taking two or more modules of mechanics, the effects of this in Higher Education and the implications for UK science, engineering and the economy, the full extent of the mechanics problem became clear and the plan for this symposium was born. Its title Newton's mechanics: Who Needs It? was a deliberate challenge. Was there a problem? Did it matter? The people who responded were in no doubt; ways of enhancing the uptake of mechanics at ages 16-19 need to be found!

Executive summary

Through a series of presentations and discussion groups the symposium explored:

- **the value of Newtonian mechanics for the teaching and learning of mathematical modelling and problem solving skills;**
- **how changes to A level Mathematics have resulted in a reduction in the level and amount of Newtonian mechanics students are now learning at A level;**
- **the impact of this reduction on students' preparation for university;**
- **practical measures for increasing the uptake of mechanics at ages 16 – 19.**

A number of recommendations, many of which could be implemented immediately, arose from the breakout group discussions stimulated and informed by the presentations. The key recommendations are given below.

---

1 The importance of Newton's mechanics

The symposium was set in context by the personal views of speakers with a concern for the UK's contribution to science and engineering. Sir Peter Williams spoke of the importance of Newton's mechanics to the UK economy. Dawn Ohlson of Thales Aerospace, outlined its place in a career in mathematics and engineering. Sir David Wallace stressed its importance as the foundation on which so much later development in science and engineering was built.

Key recommendation 1

Careers

Universities and employers should build upon links with schools and colleges to emphasize that degrees which develop mathematical modelling and problem solving skills, based on a foundation of Newtonian mechanics, can lead to stimulating and rewarding careers.

---

2 The mechanics curriculum in decline

Against a broad outline of AS/A level Mathematics development over the last fifty years, a detailed examination was presented and discussed of the effect of recent AS/A level Mathematics curriculum changes in schools and colleges.

There has been a significant decline in the take-up of mechanics following curriculum changes in 2004 and students in 30-40% of schools and colleges now have minimal access to mechanics modules (at most one).

Key recommendation 2

Student information

Schools and colleges should ensure that students intending to study physics or engineering in Higher Education know that:

(i) the content of mechanics units within AS/A level Mathematics provides important background knowledge for such university courses; and

(ii) studying mechanics is an excellent way to develop mathematical modelling and problem solving skills.

Key recommendation 3

Teacher training

Courses and resources in mechanics should be provided:

(i) within initial teacher training for all secondary mathematics and physics teachers; and

(ii) for continuing professional development of practising mathematics and physics teachers.

3 The impact on Higher Education of the decline in mechanics studied at A level

A common picture of a growing problem is emerging from studies in university departments of engineering and physics in the UK and across Europe, the situation apparently more acute in the UK.

Key recommendation 4

Universities' expectations

University departments of physics, engineering and mathematics should specify how much mechanics they would like prospective students to have studied.

---

4 Enhancing the take-up of Newtonian mechanics at ages 16-19

Encouraging action is being taken to bring further mathematics in reach of all school students. There are initiatives at work in universities. The introduction of the Engineering Diploma also has the potential to improve students' knowledge of mechanics.

Key recommendation 5

Access

Schools and colleges offering A level Mathematics should provide access to tuition in both Mechanics 1 and Mechanics 2, either as part of A level Mathematics or as part of AS/A level Further Mathematics.

Schools and colleges that cannot provide such tuition directly should collaborate with other local schools and colleges, or with external providers, to ensure their students can access tuition.

---

5 Discussion groups

Discussion groups took place after the input from the speakers and fed into the key findings and recommendations related to each of the themes above. In four groups over two sessions the groups considered the questions:

What constitutes an adequate preparation in Newtonian mechanics for students entering Higher Education in engineering, physics and applied mathematics?

How can teachers and students be supported in studying Newtonian mechanics, both at school/college and university levels?

What recommendations can we make to improve new undergraduates' mathematical modelling and problem solving skills?

The discussions also covered review and comment on demonstrations and resources in use and under development.

Key recommendation 6

Resources

Funding should be made available to create innovative and inspirational mechanics resources and integrate them into the teaching and learning of mechanics on both sides of the school/college – university interface.

The emphasis of these resources should be to help develop students' modelling and problem-solving skills.

Findings and recommendations

Findings and recommendations are presented within the report as they relate to each theme. They emerged from question and answer sessions and the later group discussions and are summarized on page 37 as a stimulus for action towards tackling the mechanics problem.

1 The importance of Newton's mechanics

There was no doubt that Newton's mechanics was now occupying a smaller place in the school curriculum and the trend showed no evidence of reversing, but how important was that? The symposium was set in context by the personal views of speakers with a concern for the UK's contribution to science and engineering.

Speaking a couple of months before the global financial crisis brought about by the credit crunch, Sir Peter Williams and Sir David Wallace focused, quite independently, on the importance of mathematical modelling skills in business. Post September 2008 the need for sound mathematical modelling skills in business has become even more apparent.

Newton's mechanics and the UK economy

To answer the symposium's fundamental and provocative question, *"Newton's mechanics: who needs it?"* Sir Peter Williams said he would not open the discussions with "paeons of praise for Newton and the mechanics that sent men to the moon". He referred instead to the London Stock Market. Dramatic movements in the price of oil and the problems of global banking were his focus. The banks were supposed to lend and borrow money. "That's what they do: only they're not doing it at the moment." If, however, only three or four of the FTSE 100 list of top UK companies were engineering companies, if no more than sixteen of them depended on the STEM disciplines of science, technology, engineering and mathematics, why should there be any concern about the significant decline in UK students' access to and knowledge of Newtonian mechanics?

The reason was clear. The people who should be driving developments in the back offices of City finance houses and wielding high level influence in industry and commerce, were the people whose creativity and imagination was grounded in the mathematical modelling and problem solving skills derived from Newtonian mechanics. These skills, which were the basis of careers in engineering or physics, could be very widely applicable. To give an example from his own career, Sir Peter referred to Oxford Instruments, where as Director he had been responsible for putting together the team that turned Peter Mansfield's Nobel prize-winning concept into the reality of the MRI scanner. Finite element mechanical engineers, cryogenic physicists, radiologists, medics and computer scientists, a diverse group of professionals trained in the STEM disciplines, together had produced a revolution in healthcare. Britain had a proud heritage in industries which intimately depended on STEM-based subjects.

It would be wrong to think, though, that if UK industry were desperate for engineers, mathematicians and scientists, supply would follow demand and there need be no cause for concern. Cause for serious concern was demonstrated through a current example. The UK engineering design consultancy, W S Atkins, largest in its field in Europe, third largest in the world, had the previous week announced a good rise in profits, but a small note in the report said that the company remained "resource, rather than market restrained". The number of engineers it needed to employ was more than the entire output of UK civil engineering graduates. The solution for this and other similarly constrained UK companies was to base activities in other countries which had invested in these qualifications.

Tackling a problem which was endemic and deep seated meant starting with the young. The review published that month by the DCSF[1] had led to government funding of £200 million to provide a specialist mathematics teacher for every primary school, up-skilling existing staff through a programme of continuing professional development. It would enable a future generation to leave school confident with mathematics. Meanwhile there was important work being done in mechanics to encourage and inspire young people. There was an urgent need to build on this, for example, on the work of MEI and the Further Mathematics Network, to develop measures for meaningful change and elevate the competences of 18 and 19 year-olds to become the technologists and scientists of the future.

a career in mathematics and engineering

The recruitment of suitably qualified graduates was a concern for Dawn Ohlson, as Director of Education for Thales, a leading UK supplier of information systems for the aerospace, defence and security markets. The nature of the business required her to select from predominantly British and EU candidates and there were not enough of them.

A description of her own career in Thales showed what exciting opportunities were available when mathematics was applied to challenging real world situations. Search and rescue steering algorithms for the Sea King helicopter, for example, demanded an understanding of trigonometry at around A level standard to work out the turn direction, the patterns for maintaining a creeping line ahead pattern or making a sector search, taking into account different wind or tide conditions and the mechanics of the helicopter. Similar mathematics was needed to calculate sonobuoy drop patterns and to calculate how to steer a helicopter to turn into wind and land on an aircraft carrier.

Different sets of challenges and different kinds of mathematics but a similar need for precision were involved in handling programmes on bid management where a small error could have huge implications. Yet other different uses of higher level mathematics were to be found in the research and technology department, an example being shown in work on modelling speech production.

With a growing need for recruitment in this field there was a fear that if the mechanics problem were not sorted out, talent in the UK would be lost. The engineering industry urgently required more engineers if it was to survive. It was vital to find ways to inspire young people to take up the mechanics they needed to lead on into an engineering career.

[1] Sir Peter Williams, Independent Review of Mathematics Teaching in Primary Schools and Early-Years Settings, (DCSF, 2008) http://publications.teachernet.gov.uk/eOrderingDownload/Williams%20Mathematics.pdf

Newton's mechanics in Higher Education

Mechanics in physics and engineering

Confirming the importance of Newtonian mechanics as the foundation of further studies in physics, Sir David Wallace referred to his own education, where he had worked in theoretical particle physics. While there were areas of the physical world where Newtonian mechanics as an approximation broke down, where quantum mechanics for the very small, or relativity for things approaching the speed of light, took over, Newtonian mechanics remained the foundation. To see how quantum mechanics or relativity had to be formulated required building on Newton's equations to have sound starting points for the more general theories. A grounding in Newtonian mechanics was fundamental.

Similarly, its importance for underpinning everything in engineering was pervasive. You could not work on fluid dynamics or fluid flow without starting from basic Newtonian principles. Subjects might appear to have left Newtonian mechanics behind but it remained the foundation and had applications beyond the disciplines of physics and engineering.

Transferable skills

As part of his role as Director of the Isaac Newton Institute for Mathematical Sciences, Sir David had begun to interact with people in the City. He had recently met a global banker, a major world-wide employer in retail and commercial banking, who had commented that he was desperate to "increase the simulation and modelling" for his own banking business. "I want mathematicians, I want physicists, I want hard engineers. I'll put them through the business but that's the skills I want." To illustrate why, he had described how during a banking collapse when other banks lost heavily, his did not because their model scenarios had enabled them to think through rapidly the likely impact and potential future for that situation.

Such opportunities should be embraced with enthusiasm and students made aware of them. The breadth of the impact and value of these skills learned through Newtonian mechanics should not be underestimated.

Teaching

With regard to the teaching of mathematics and physics in universities, fifteen years ago the age profile had suggested there would be a crisis. It had not materialised because approaching 50% of recruiting into universities was now from abroad. That might mean a vibrant higher education system, or it might give cause for concern. The situation was very different in schools, as shown in a report on the shortage of physics teachers in schools[2] given national news coverage the previous day.

At the 4th Ogden Science Forum at the Møller Centre two weeks earlier,in response to a teacher's question, "How do we inspire the kids?" Bill Bryson's answer had been, "Think back to what inspired you". Students' attitudes and competences might have moved on but it is the enthusiasm of their teachers which will inspire them.

[2] Alan Smithers, Pamela Robinson, Physics in Schools IV: Supply and Retention of Teachers, (Gatsby Charitable Foundation, Carmichael Press University of Buckingham, 2008)

Findings and recommendations

Newtonian mechanics has a vital role to play in the education of potential mathematicians, physical scientists and engineers; it

- introduces important concepts,
- provides ample scope for applying mathematics to solving real physical problems,
- develops students' mathematical modelling and problem solving skills,
- improves students' fluency in algebra and calculus.

Mathematical modelling and problem solving skills are generic skills which, once mastered, prepare students for problem solving in all branches of science and engineering. These skills are also used by creative thinkers in industry and in business and commerce.

The ability of teachers to inspire young people with their own enthusiasm for the subject can play a vital role in encouraging take-up of the subject, by staff as well as students.

Key recommendation 1

Careers

Universities and employers should build upon links with schools and colleges to emphasize that degrees which develop mathematical modelling and problem solving skills, based on a foundation of Newtonian mechanics, can lead to stimulating and rewarding careers.

2 The Mechanics Curriculum in Decline

Roger Porkess presented an overview of the position of mechanics in the A level curriculum over the past 50 years, showing how A level Mathematics has evolved over this period in response to new technology, changes in employment patterns and the increasing proportion of students continuing education post 16. The introduction of Curriculum 2000 resulted in a 20% reduction in the number of students taking A level Mathematics. In response the A level Mathematics curriculum was revised for 2004, with the effect that applied mathematics was reduced by one third. Numbers are recovering as a result of the 2004 changes, but fewer students now take more than one mechanics module.

Sue de Pomerai then gave a detailed analysis of the impact on mechanics entries across the 2004 changes to the present day.

Developments in A level Mathematics over the last fifty years

Roger Porkess

This presentation was illustrated by A level examination papers from 1960, 1984, 1992 and 2007[3]. These papers were the manifestations of deeper changes that were happening in society, education in general and the perception of the role of mathematics.

A levels began in 1951. A level Mathematics then contained roughly equal amounts of pure mathematics and mechanics; mechanics was compulsory. To today's students the **1960** paper would look somewhat forbidding with imperial units and no diagrams. However, most of the questions were actually very similar to current questions in what they asked and in the steps used to lead candidates through them, although this is now done more explicitly by dividing them into separate parts. In contrast to present practice, candidates were allowed a choice of questions. As now, Further Mathematics was available as an alternative to the single subject. A major difference was that in those days almost all mathematics students took physics and chemistry as their other two A levels and the content of these reinforced their mathematics.

The next examination paper, from **1984**, was set in quite a different environment. With the launch of Sputnik, the Russians won the space race and this set off a wave of curriculum development in the United States. This spread across the Atlantic, causing serious questioning of the mathematics curriculum, including a debate about the nature of mechanics, whether it

[3] Sample A level examination papers 1960-2007 www.mei.org.uk/files/pdf/mechanicsexams.pdf

should be taught as an axiom system or as a means of modelling the physical world. Another major influence was the invention of calculators; in mathematics these moved the balance of perceived importance away from calculation and towards understanding and interpretation.

The 1984 paper was a specimen for one of the new syllabuses that followed the introduction of the first common core. These syllabuses were not successful. Their content proved to be heavy for the time available and, with a much wider choice of subjects now available, the number of students taking A level Mathematics fell steadily. Alongside compulsory pure mathematics students now had a choice of statistics and mechanics questions. Many opted for statistics, finding it easier and relevant to them. One informal survey of 500 papers showed that, where there was a choice, 93% of the questions answered were statistics and only 7% mechanics.

A major change occurred in 1990 when teaching began on the first modular A level, MEI Structured Mathematics. This development had been motivated by two concerns: to stem the decline in students taking mathematics post 16 and to ensure that mechanics was not lost. Both aims were achieved. The **1992** paper was one of the earliest set for a modular syllabus and it looked rather different from previous mechanics papers. It was a Mechanics 1 paper and so, with separate papers to follow for Mechanics 2 and 3, it was shorter, with just 4 questions. Greater use was made of diagrams so that candidates could concentrate on the mechanics and there was more emphasis on modelling and interpretation.

These developments were influenced by the Gatsby-funded *Mechanics in Action* project. Its effects should not be underestimated and the people involved in it, including Mike Savage and Geoff Wake at this symposium, were to be congratulated on their success in stimulating interest and enthusiasm. This period was a high point in school mechanics.

The next major change came with Curriculum 2000. In view of the problems that this was to cause, it is paradoxical that there was actually very little change to the mathematics syllabus. It was changes elsewhere that undermined mathematics. Most students were now to take four or five subjects instead of their previous three, and they were expected to complete much more mathematics in much less time. Unsurprisingly there was a very high failure rate, nearly 30%, in AS level Mathematics in 2001 and many students gave up mathematics. This in turn led to a reduction in A level numbers of nearly 20% the following year.

To rectify this a new syllabus was devised for first teaching in 2004 with the pure mathematics content spread over four modules rather than three and applied mathematics reduced from three modules to two. The **2007** paper was an example of Mechanics 1 in this syllabus.

Overall the 2004 syllabus is working well. The number of students taking A level Mathematics has recovered to pre Curriculum 2000 levels and is still increasing.[4] However, the reduction from three applied modules to two means that many students do at most one module of mechanics, and the amount of mechanics being learnt as part of A level Mathematics is at an all-time low, the more so since no syllabus now includes any mechanics coursework.

The situation is not, however, as bleak as this. To reach any depth in mechanics students now need to take Further Mathematics and the growth and success of the Further Mathematics Network can ensure that all students have access to it. The total content of single A level Mathematics plus AS level Further Mathematics equates to the content of A level Mathematics in 1980, so offers an achievable and desirable outcome.

---

4 Stephen Lee, Roger Porkess, An investigation into the increase in C1 candidate numbers in January 2008, (MEI, 2008)

AS/A level Mathematics: the impact of recent curriculum change in schools and colleges

Sue de Pomerai

(See Appendix 1 for more detail on the structure of AS/A level Mathematics and Further Mathematics)

With a focus on the period from 2000 to 2007, a close look at these curriculum changes shows how they have affected the decisions made about applied mathematics in schools and colleges. Data from the major awarding bodies reveals trends in the take-up of mechanics and the opportunities for students to study it.

The changes to AS and A level Mathematics from 2000 must be viewed in relation to other changes in post-16 education in the 1990s. The development of many more A level subjects meant that mathematics was competing for students with many newer subjects, such as psychology and media, and was no longer an automatic A level choice, even for those who were very able. In addition the Government decision to ensure that more students continue to study post-16 has led to many previous school leavers now going on to A level, some choosing to study mathematics at AS or A level. Entry to A level courses must be seen to be a level playing field and many schools and colleges set an entry level at Grade C GCSE, though most mathematics teachers hold the opinion that a Grade C at GCSE is not adequate for entry onto an AS or A level Mathematics course.

Results of the changes

2005 was the first year of the new specifications. The majority of teachers are very happy with the current system; students have more time to study, so achieve a better grasp of the underlying pure mathematics. Strategic decisions can be made about which combinations of applied modules will give greatest overall benefit to their students. Since A level Mathematics includes only two applied modules, in order to maximise grades most schools/colleges opt to do M1/S1 or M1/D1 or S1/D1. This makes perfect sense for them since it optimizes students' grades, a very important consideration in a league table-led education system! Additionally, given the wider range of ability and interests of students taking A level Mathematics, it introduces a range of applications. The unfortunate negative effect of this is that, unless they are also taking a Further Mathematics qualification, students rarely have the opportunity to study any one area of applied mathematics in depth and some individual students may not get access to the most appropriate A2 applied modules.

The data in the table below from the awarding body Edexcel shows the combinations of mathematics modules taken by students in 2006

Optional modules	Number of candidates	% total candidates	% A grade
Mechanics 1 (AS) Mechanics 2 (A2)	4,328	21.8	52.5
Statistics 1 (AS) Statistics 2 (A2)	4,012	20.2	42.2
Decision 1 (AS) Decision 2 (A2)	190	1.0	42.6
Mechanics 1 (AS) Statistics 1 (AS)	8,970	45.1	47.1
Statistics 1 (AS) Decision 1 (AS)	1,433	7.2	38.8
Mechanics 1 (AS) Decision 1 (AS)	944	4.7	50.1

Table 1: applied mathematics modules studied by students in 2006

These data show that 57% of students did not study any applied mathematics modules at A2 level and only 21.8% of students took M2, many of whom would be Further Mathematicians. The figures did not distinguish between those taking only A level Mathematics and those also taking Further Mathematics at either AS or A level, but the conclusion could be drawn that many students not studying at least AS level Further Mathematics do not have the opportunity to take M2 (or S2/D2 for that matter).

Trends in applied mathematics modules

The data in Table 2, which has been collated from the major English awarding bodies, shows the entries for applied mathematics modules from 2001 to 2007.

	2001	2002	2003	2004	2005	2006	2007
M1	67040	70910	68160	69923	54724	61826	65935
M2	11462	26133	27328	28155	21196	18088	19110
M3	1331	6949	6851	6770	6021	4358	4482
M4	341	1921	1960	2131	2213	914	849
S1	85725	90603	88995	89905	76321	82263	89192
S2	10646	28898	30220	30906	21927	18140	19201
S3	1008	6602	6108	5619	3296	977	973
S4	31	1976	2122	2403	1978	291	273
D1	15455	25121	28524	29262	23038	27190	30745
D2	1024	4684	5475	6311	3399	3345	3674

Table 2: entries for applied mathematics modules from 2001 to 2007

Using those data the figure below compares the numbers taking level 1 modules of statistics, decision mathematics and mechanics from 2001 to 2007.

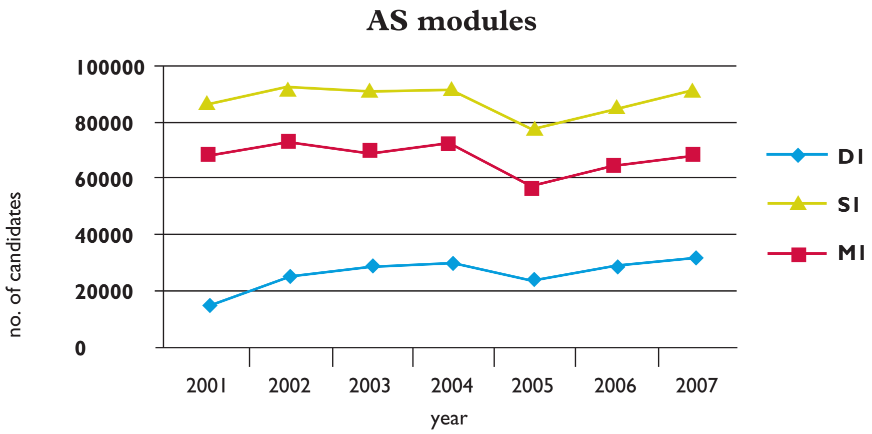


Figure 1: trends for AS level applied mathematics modules 2001-2007

After a dip in 2005, due to the reduction from three to two applied modules at A level, M1 and S1 numbers are recovering in line with the numbers taking A level Mathematics. M1 is approaching the numbers taking it in 2002.

The figure below compares the numbers taking A2 level applied mathematics modules over the same period.

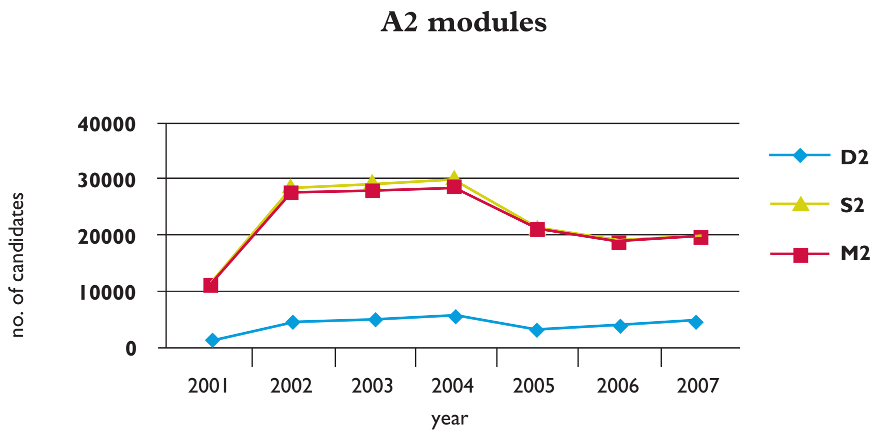
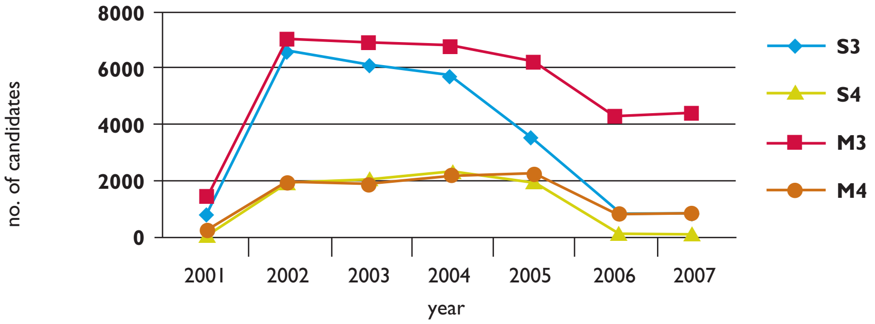


Figure 2: trends for A2 level applied mathematics modules 2001 to 2007

It should be noted that the figures for 2001 are not a true reflection, being the final year of the pre Curriculum 2000 specifications. There was a big rise in the numbers taking M2 and S2 in 2002, the first year of Curriculum 2000, when all students were required to take at least one applied module at A2 level. Numbers reached a high point in 2004, the last exams for that specification. The current specifications were introduced for teaching from 2004. Since September 2004 students can only study two applied modules as part of their A level, both of which could be AS level modules, so M2, S2 and D2 numbers have declined because most students no longer do them as part of their A level. Around 29% of students who took M1 continue on to M2, most of whom will be taking AS or A level Further Mathematics.

The figure below compares the numbers taking higher modules of statistics and mechanics over the same period.



Level 3/4

Figure 3: trends for higher level applied mathematics modules 2001 to 2007

The effects of the 2004 reforms can be seen clearly in the sharp decline in numbers taking M3 and M4 in 2006, since students will take these higher modules in the second year of study. Interestingly, the numbers taking M3 and M4 have not declined as sharply as those taking S3 and S4. The 2007 entries appear to show a slight rise which is due to the increased number of students taking Further Mathematics.

Opportunities for access to mechanics modules

The data from Edexcel given in Table 3 below shows figures for 2007 of the number of Centres offering the range of mechanics modules.

	No of Centres with candidates
Mechanics 1	1332
Mechanics 2	828
Mechanics 3	414
Mechanics 4	146
Mechanics 5	61
A level Mathematics	1419
A level F Mathematics	635

Table 3: figures from one awarding body for the number of Centres offering the range of mechanics in June 2007.

The following important points can be noted

- Of the 1419 Centres, only 1332 were offering M1, so students in 87 Centres did not have access to any mechanics modules.
- It can be seen that only 828 out of the 1419 (58.4%) Centres, are teaching M2 to their students.
- Most, if not all, the Centres that offer A level Further Mathematics will offer M2 so the implication is that fewer than 200 Centres are offering M2 as a module in A level Mathematics.

Factors affecting the teaching of mechanics in schools and colleges

The November 2007 QCA report, *Evaluation of participation in GCE Mathematics*[5] refers to the 'Smith paradox' as

> 'the A level being designed both for those who need mathematics as an underpinning for other areas of study and for those who are able mathematicians who will progress.' [The research shows that] 'mathematics students are a highly achieving sub-group of A level students. The issue, then, is whether we are content for this to continue to be the case? Are we content to draw in more high achievers to study mathematics – increasing the clever core? Or is the discussion relating to providing a more accessible qualification, one that implies that mathematics at A level should be embracing a broader range of ability, more in line with other A levels?'

Argument over the perceived difficulty of mathematics, physics and chemistry relative to other A levels was further stimulated by the report issued on the morning of the symposium by the University of Durham[6] indicating these subjects were one grade harder.

Teacher training

There is a shortage of good mathematics teachers and an even greater shortage of mathematics teachers who can teach mechanics. Recent research[7] indicates that around 30% of those teaching mathematics in secondary schools have no mathematics qualifications beyond A level. The Adrian Smith report, *Making Mathematics Count,* identified provision of well qualified teachers as a problem and made strong recommendations for the continuing professional development of existing teachers. However, many experienced mechanics teachers are approaching retirement and younger teachers are less likely to have mechanics specialism, so this problem is likely to increase.

---

5 Evaluation of participation in GCE Mathematics, QCA (2007) available online at: www.ofqual.gov.uk/719.aspx

6 Relative difficulty of examinations in different subjects (University of Durham, 2008) http://www.cemcentre.org/documents/news/SCORE2008summary.pdf

7 The UK's science and mathematics teaching workforce, (Royal Society, 2007) http://royalsociety.org/downloaddoc.asp?id=4867

Laura Kounine, John Marks, Elizabeth Truss, The value of mathematics, (Reform, 2008) http://www.reform.co.uk/documents/The%20value%20of%20mathematics.pdf

Findings and recommendations

Currently 30-40% of schools and colleges enter their students for at most one unit of mechanics. As a result, many students arrive at university to study physics, engineering and mathematics having studied at most one unit of mechanics.

The reduction in the amount of Newtonian mechanics studied by many students as part of A level Mathematics has resulted in a decline in students' modelling and problem solving skills.

Key recommendation 2

Student information

Schools and colleges should ensure that students intending to study physics or engineering in Higher Education know that:

(i) the content of mechanics units within AS/A level Mathematics provides important background knowledge for such university courses; and

(ii) studying mechanics is an excellent way to develop mathematical modelling and problem solving skills.

Teachers are generally happy with the current structure of AS/A level Mathematics and Further Mathematics.

Recommendation 2.1 *AS/A level Mathematics and Further Mathematics* Short term changes to the current structure of AS/A level Mathematics and Further Mathematics before the 2013 review should be avoided.

Recommendation 2.2 *AS/A level Mathematics* A level Mathematics should remain at 6 modules to avoid the risk of reducing the amount of mechanics available to students and fragmenting the applied mathematics curriculum.

There is a shortage of mathematics teachers with the skills to teach mechanics as part of A level Mathematics or Physics. CPD in mechanics is not readily available and there are difficulties for schools in finding cover for teachers to attend.

Key recommendation 3

Teacher training

Courses and resources in mechanics should be provided:

(i) within initial teacher training for all secondary mathematics and physics teachers; and

(ii) for continuing professional development of practising mathematics and physics teachers.

3 The impact on HE of A level mechanics decline

Parallel studies carried out by Stephen Lee at Loughborough University and Dick Clements at the University of Bristol revealed evidence of students' needs for help in mechanics before and after 2004 and suggested measures academics need to take to address them.

Widening the focus to the situation across Europe, Gareth Jones reported on a study he was leading for the EUPEN (European Physics Education Network) group of 160 universities on the transition from school to university study in physics. As the study nears completion it is revealing a Europe-wide problem, but one which appears most acute in the UK.

Measuring the mechanics problem in UK universities

Loughborough University Stephen Lee

Background
It had come to light that a number of students visiting the Mathematics Learning Support Centre in 2002 and 2003 were seeking help with their first year mechanics work. Talking to them it was seen that some had not studied, or some had not even had the opportunity to study mechanics modules during their A level studies. This merited further investigation and research into it was begun at Loughborough University in 2003[8].

The research comprised three key areas: the schools' perspective, the undergraduate students' perspective and the university academics' perspective. Some findings from the first two of these are presented here.

Schools' perspective
Two key questions that were considered in this area were:

1. What is the actual availability of applied modules for students to study in Mathematics A levels?

2. What is the uptake of these applied modules?

In an ideal world such data would be available from awarding bodies, but the reality was that a complete dataset was not available during the life-time of the project. Consequently, a large scale survey was created to gain an understanding of the situation.

Surveys were posted to a sample of 497 schools/colleges (approximately one fifth of those that offer A levels) in both January 2004 and January 2006.

8 Stephen Lee, 'The Repercussions in Higher Education of the Changes in the Teaching and Learning of Mechanics in Schools in England' (unpublished doctoral thesis, Loughborough University, 2006)

This was to take account of the changes to the A level Mathematics specifications in September 2004. There were 243 replies from schools in 2004 and 225 replies in 2006; this encapsulated some 13,754 AS/A level mathematics students in 2004 and 13,673 in 2006.

Table 1, which shows the percentage of schools who offered a given number of mechanics modules in 2004 and 2006, clearly indicates there to be a decline in the availability of two or more mechanics modules in the sample (from 74% of schools offering two or more modules, to 60% in 2006). The 2004 figures relate to availability in both A level Mathematics and Further Mathematics. The 2006 survey was modified so that it could be indicated if the availability was only, for example, to further mathematics students.

Mechanics	0 modules available	1 module available	2 or more modules available
2004	5	21	74
2006	8	32	60

Table 1 – % of schools offering a given number of mechanics modules in 2004 and 2006

In Table 2 it can be seen that the availability of two mechanics modules (and two statistics modules as well) is greatly reduced when looking at only the A level Mathematics qualification. This highlights one of the positive aspects of further mathematics, that it greatly increases the opportunity to do more applied modules. Indeed, an AS in Further Mathematics gives students the chance to double the number of applied modules that a student would have studied if they only completed A level Mathematics.

A level only	0 modules available	1 module available	2 modules available
Mechanics (2004)	16 (8)	46 (32)	38 (60)
Statistics	11	52	37
Discrete	64	28	8

Table 2 – % of schools offering a given number of modules in A level Mathematics in 2006

Table 3 (overleaf) looks to answer the second question, regarding uptake of applied modules. There it is evident that although there appeared to be little change in the percentage of students who studied the first level module in 2004 and 2006, the percentage was considerably lower for the second level modules in 2006. 18% of students in the 2004 sample studied the second mechanics module, whereas only 13% of the 2006 sample studied the second mechanics module. This raises concerns, but indicates an effect of a change in specification in 2004. For an A level in Mathematics students would now be more likely to study two first level applied modules, rather than study the first and second level of one strand. (This is supported by QCA's analysis[9].)

[9] QCA (2007) Evaluation of participation in GCE Mathematics, Available online at: www.ofqual.gov.uk/719.aspx

	1st Module		2nd Module	
	2004	2006	2004	2006
Mechanics	**42**	**42**	**18**	**13**
Statistics	51	52	19	12
Discrete	17	19	3	3

Table 3 – % of students who studied the 1st and 2nd modules in 2004 and 2006

Undergraduate students' perspective

While the findings from the questionnaire to schools gave some cause for concern, the evidence of university students' difficulties with mechanics also needed investigation. Several interventions occurred, including surveying students on their prior studies, asking students to complete a mechanics diagnostic test (in addition to a mathematics one), considering how their first year performance related to such factors and interviewing students. Only the first of these, ie the survey findings, is given here. Findings from the others can be seen in Robinson et al (2005)[10] and Lee et al (2006[11], 2007[12], 2008[13]).

As can be seen from Figure 1, approximately 68% of the 1,087 engineering students surveyed had studied two or more modules of mechanics. This is considerably more than those in schools/colleges in 2004 (and 2006) where, at most, 26% (and 17% respectively) of students had studied two or more mechanics modules. This appears to paint a less worrying situation, but the fact remains that around a third of university students in the sample had studied little or no mechanics. In a typical group of, say, 150 first year students, it is certainly an issue if 50 of them have a lack of prior knowledge.

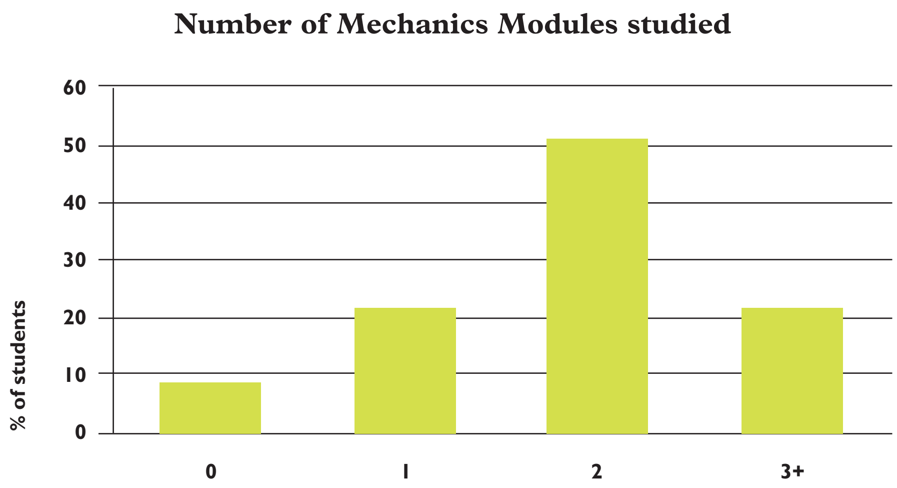


Figure 1 – % of 1087 engineering students who had studied a given number of mechanics modules

[10] Robinson, C. L., Harrison, M. C. and Lee, S. (2005) The Mechanics Report – Responding to the Changes in the Teaching and Learning of Mechanics in Schools. Commissioned by the UK Higher Education Academy Engineering Subject Centre. Available online at: www.engsc.ac.uk/downloads/resources/mechanics.pdf

[11] Lee, S., Harrison, M. C. and Robinson, C. L. (2006) UK engineering students' knowledge of mechanics on entry: has it all gone? In W. Aung, et al (eds). Innovations 2006, World Innovations in Engineering Education and Research, Ch 20, INEER/Begell House Publishing, Redding, USA pp 247– 255

[12] Lee, S., Harrison, M.C. and Robinson, C.L. (2007). Recent changes in A level Mathematics: is the availability and uptake of mechanics declining yet more? Teaching Mathematics and Its Applications 26(3), pp154 - 166

[13] Lee, S., Harrison, M.C., Pell, G. and Robinson, C.L. (2008). Predicting performance of first year engineering students and the importance of assessment tools therein Engineering education – The Journal of the Higher Education Academy Engineering Subject Centre, 3(1), pp 44 – 51.

In conclusion

The findings that have been detailed here, which are part of a much larger analysis set, support the idea that concern should be shown over the availability and uptake of mechanics modules in mathematics A levels. It also emphasizes that students' knowledge of mechanics at the start of degree level courses is naturally dependent on what they have studied at school or college. Some students will have little knowledge, while others may have a considerable amount. This presents a challenge to university academics who expect a level of prior knowledge of mechanics that can no longer be assumed and who need to be brought up to speed with pre-university developments.

University of Bristol Dick Clements

In parallel with the work being done in Loughborough, work at the University of Bristol came independently to very similar conclusions. The University was facing the same problem. Staff were not fully aware of the changes that had happened in schools and colleges in the period since they had been studying at school. No longer was applied mathematics all about Newtonian mechanics. Now it covered the three areas of mechanics, statistics and decision mathematics and first year university teaching had to cope with the way this had affected students' lack of preparation for studying engineering.

This research was entirely based upon the experience of students who had been admitted to study engineering at Bristol University. A level Mathematics had gained in breadth but lost in depth and this had affected teaching and led to teaching inefficiencies. Staff had previously expected students to come well prepared in mechanics but not to know much about probability and statistics. Now, some students were bored by work they had to do on statistics but out of their depth with the level of mechanics they were assumed to know.

The point of this work was to try to put figures on these observations. Mechanics was the focus but at the same time data were collected that would enable similar statements about probability and statistics and decision mathematics. The intention was to find out the effect of the change between the 2000 and the 2004 syllabus, including the parallel changes in the areas of statistics and decision mathematics.

Students entering all the engineering degree courses in October 2005 and October 2006 were surveyed. It was a computer-mediated survey delivered to the students during their first two days in the faculty so there was a nearly 100% response to the survey. The students entering in 2005 had obviously been taking their A levels under the 2000 regime. Those in 2006 had mostly taken theirs under the 2004 regime. The data were collected anonymously, aggregated into departments. Students who had taken an alternative qualification to A levels (eg European Baccalaureate) were taken out of the analysis.

2005	n	None	M1	M2	M3	M4	M5+
Aeronautical	54	11 (20%)	43 (80%)	34 (63%)	9 (17%)	3 (6%)	1 (2%)
Civil	55	2 (4%)	53 (96%)	36 (65%)	9 (16%)	5 (9%)	1 (2%)
Electrical	47	7 (15%)	40 (85%)	32 (68%)	10 (21%)	5 (11%)	2 (4%)
Mechanical	83	6 (7%)	77 (93%)	62 (75%)	23 (28%)	7 (8%)	3 (4%)
Eng Mathematics & Design	46	4 (9%)	42 (91%)	34 (74%)	14 (30%)	10 (22%)	3 (7%)
Comp Sci & CSE	47	7 (15%)	40 (85%)	37 (79%)	12 (26%)	4 (9%)	1 (2%)
Overall	**332**	**37 (11%)**	**295 (89%)**	**235 (71%)**	**77 (23%)**	**34 (10%)**	**11(3%)**

Table 1 Mechanics 2005 University of Bristol Faculty of Engineering

There was not much systematic difference between the departments which was quite encouraging. The figures in the 'Overall' row across the bottom proved to be very similar to the figures that were published by Stephen Lee and Carol Robinson et al in the intermediate report from the Loughborough survey. They show 11% of students having taken no mechanics, 89% taking one module, 71% having taken two modules and then falling very rapidly at that point. The break point was between M2 and M3, falling to 23%, then 10% and 3% for M4 and 5+.

The picture was similar in 2006, but the figure which did change was at M2.

	None	M1	M2	M3	M4	M5+
2005	11%	89%	71%	23%	10%	3%
2006	11%	89%	58%	21%	5%	0%

Table 2 Mechanics 2005 and 2006 University of Bristol Faculty of Engineering

A comparison of the overall figures for 2005 and 2006 shows that the number of students who had taken none or just one mechanics module was exactly the same, those taking M3 and M4 were almost the same, but the number taking M2 showed a serious change. There having been no connection with the Loughborough study, it is interesting to see that these figures, 71% down to 58%, are almost the same as Stephen Lee had found in his work. A common picture is appearing.

None	S1	S2	S3	S4	
2005	25%	75%	32%	8%	2%
2006	26%	74%	38%	11%	3%

None	D1	D2	
2005	71%	29%	6%
2006	72%	28%	5%

Table 3 Statistics and OR University of Bristol Faculty of Engineering

A comparison of figures for 2005 and 2006 for the statistics and the decision mathematics modules, again shows decision mathematics with almost identical figures pre and post the syllabus changes, and statistics with almost identical figures except that the figures for those taking two statistics modules have gone up very slightly compared with the 2000 syllabus.

M1 M2	M1 S1	M1 D1	S1 S2	S1 D1	D1 D2
37%	52%	6%	0%	6%	0%

Table 4 Relative Popularity of Choices University of Bristol Faculty of Engineering

Finally, the data show the relative popularity of the different combinations of choices, bearing in mind these are restricted to students who are studying engineering. The most common combination is the first level module in mechanics and the first level module in statistics, followed by those taking M1 and M2.

The conclusion which can be drawn from this is that the most obvious effect of reducing the number of option modules has been a further reduction in the number of students studying mechanics beyond M1. The effect on the uptake of statistics and decision modules has been by comparison negligible. This change further reinforces the conclusions of Robinson et al and earlier work from Bristol[14] concerning the decline in the level of preparedness of students entering degree courses in engineering.

Mechanics is an important enabler for proper understanding of engineering science and design analysis, yet these two independent studies show a worsening picture of students arriving at university without the background knowledge of mechanics which they need. Continuing change in the syllabus of university degree courses will be required to compensate for the lower entry skills.

[14] Clements, R.R, ‘Prior Knowledge of Mechanics amongst First Year Engineering Students’ Teaching Mathematics and its Applications, 26 (2007), 119-123.

Findings and recommendations

Universities report that many students currently admitted onto degree courses in physics, engineering and mathematics, even those with top grades in A level Mathematics, have an inadequate preparation in mathematical modelling and problem solving skills and a poor knowledge of mechanics.

Studying just one unit of mechanics, M1, at A level provides students with a basic introduction, but does not constitute an adequate preparation for physics or engineering in Higher Education.

Most university departments of physics and engineering and mathematics will prefer and some will expect prospective students to have taken two units of mechanics (M1 and M2) as part of A level Mathematics or AS/A level Further Mathematics.

Key recommendation 4

Universities' expectations

University departments of Physics, Engineering and Mathematics should specify how much mechanics they would like prospective students to have studied.

Most university departments of physics and engineering admit some students who are well prepared in mechanics and others who are ill-prepared. The latter may be seriously disadvantaged unless special provision is made for them. Diagnostic tests can signal the importance of mechanics for the degree course, alert staff and students to students' strengths and weaknesses, but must be well-designed and do require follow-up.

Recommendation 3.1 *Universities' information and update*

University departments of physics and engineering should:

(i) monitor changes in A level Mathematics and Further Mathematics and other relevant pre-university qualifications;

(ii) gain an understanding of incoming students' prior knowledge of mechanics; and

(iii) review teaching methods and styles to accommodate students with varying levels of mechanics knowledge.

Mind the gap! A transition problem across Europe

Gareth Jones

A gap in understanding basic physics concepts, an unfamiliarity with mathematical reasoning and a deficiency in problem solving was being recognized across Europe at the transition to university physics study. A two-year study into the problem by the EUPEN group of 160 European universities was nearing completion.[15] Detailed questionnaires had been returned from more than 120 universities, confirming the extent of the problem and providing insights into the causes, principally changes in school curricula and shortages of well-qualified teachers. The study also involved surveys of the experience of 1st Year students and visits to schools in several European countries to interview teachers. Looking at changes recorded over the past decade, while students' skills in other areas such as ICT were seen to have improved, the areas of sharpest decline were in mathematical skills and scientific background and knowledge.

The study group leaders joining Imperial College, London, were from the Universities of Aveiro, Portugal, Nijmegen, Netherlands, Iasi, Romania and TU Wien, Austria and school visits formed an important part of the survey. Comments from schools in those countries echo those from this country, for example: ' We have one teacher able to teach specialist physics at ages 17 and 18 and when he retires this year I don't know what we'll do.' or, 'Not enough time … Syllabus does not give the right coverage of physics topics and is too little mathematically based.' Such constraints have meant difficulties in giving careful, reasoned explanations to motivate students. Young people were told things but were not going to acquire a good understanding without opportunities for modelling and problem solving using mathematics.

The survey being still under way, national differences had not yet been studied in detail, but it was clear that in the UK the gap in mechanics and the physics teacher shortage problems were substantially more serious than in other countries.

A related and larger scale survey across more than nine disciplines, including physics, mathematics, earth sciences and chemistry was also under way in 2008, under the *Tuning*[16] initiative which aims to involve universities in determining the direction of Higher Education reform post-Bologna. Surveying employers, graduates, students and academics, one response in particular from employers was significant. Asked to rate 'general' and 'subject-specific' competences, the most highly rated general competences were: in first position 'apply knowledge in practice' and then 'identify, pose and resolve problems'. The difference between the subject-related skills most highly rated by employers, graduates and students was also interesting. While all groups rated the ability to enter new fields in top position, employers put modelling skills next, while students rated them last.

The conclusion to be drawn in particular from the EUPEN survey was that the gap between school and university in physics was widening throughout Europe. Mechanics is an important element of that gap, particularly in the UK. Universities see deficiencies in the abilities of their incoming students in: mathematical reasoning, problem solving, knowledge of and ability in

15 EUPEN Working Group 5, see http://www.eupen.ugent.be/wg/wg5.php

16 Tuning Educational Structures in Europe, http://tuning.unideusto.org/tuningeu/

mathematics, and modelling of the real world based on physics and mathematics. Now, too often, young people are not even being introduced to this area, the area which could and should be inspiring them, and so they acquire a misleading view of the nature of physics as a scientific discipline.

Findings

Response from the EUPEN group of 160 European universities reveals a widening gap in students' knowledge of mechanics at the transition to university study in physics, and that this gap is most severe in the UK. Pending publication of its report, it is anticipated that EUPEN recommendations will include improving communication between schools and universities, particularly over teacher training and collaboration over development of pedagogical materials. It will also recommend that steps should be taken to enhance the links between physics and mathematics in the school curriculum.

Note: The EUPEN recommendations are in accord with this symposium's recommendations 4.2, 4.3 and 5.2.

4 Enhancing the take-up of Newtonian mechanics at ages 16-19

There must be ways for students and teachers to gain access to mechanics and three presentations gave encouragement about how this could be done at school/college, at university and via the new Engineering Diploma. Charlie Stripp outlined the increasing take-up of AS/A level Further Mathematics and how the Further Mathematics Network is supporting teachers and students. David Abrahams illustrated a range of strategies from the experience of Manchester University and the work of the IMA. Fred Maillardet gave news of the Engineering Diploma and the possibilities it offered for making connections with real world applications. Good communication was the key to success for them all.

At school and college –the Further Mathematics Network

Charlie Stripp

Mechanics units are available to students as part of AS/A level Mathematics and Further Mathematics, but they are optional. Many choose to take M1. For engineering and science-based studies ideally they should also choose M2 and possibly M3. Unfortunately many do not, either because it is considered difficult or because schools cannot offer it.

(See Appendix I for more detail on the structure of AS/A level Mathematics and Further Mathematics)

Further Mathematics offers an answer. Since 2004 AS level Further Mathematics has changed and is no longer an elite qualification. Only the brightest A level Mathematics students used to take further mathematics. The new specification for further mathematics means that AS Further Mathematics is a qualification that is accessible to any student doing mathematics A level. It is flexible in that you can study it alongside AS/A level Mathematics in year 12, or in year 13, or across both years. Students may be undecided on their university plans when they complete GCSEs at age 16. By the end of year 12 they may be thinking of studying engineering or physics at university and can choose to take further mathematics then. It offers an intelligent choice for a student to make at that crucial point.

Three equally-weighted units make up AS level Further Mathematics. A compulsory pure unit, Further Pure 1 reinforces and extends the AS/A level Mathematics. Many university engineering and physics departments are worried about basic fluency in algebra. This unit reinforces students' skills and introduces complex numbers and matrices, which do not appear in AS/A level Mathematics. The other units are optional and can both be mechanics. They could be M1 and M2, or students doing M1 as part of their mathematics A level could take M2 and M3, or a variety of other options. Taking AS Further Mathematics gives students more access to mechanics.

Our experience suggests that further mathematics improves students' A level Mathematics grades by reinforcing their skills and helping them to develop greater fluency in algebra. It offers a very good preparation for the transition to mathematics-related studies at university.

Following a successful 4 year pilot, funded by the Gatsby Charitable Foundation, the Further Mathematics Network (FMN) began, with government funding, in academic year 2004/5. Since then, A level Further Mathematics entries have risen by 39% and AS level Further Mathematics entries by 89%[17]. The FMN has greatly increased accessibility. Now any school or college in England unable to offer further mathematics tuition to students can do so through the FMN. Universities had been claiming it would be discriminatory to encourage students to take further mathematics qualifications if they had no access to tuition. That situation has changed. The FMN means that further mathematics is available to any student with the motivation to take it. The FMN supports schools and colleges to offer further mathematics by providing resources, expertise and help for teachers. Where schools and colleges are unable to provide tuition themselves, the FMN can provide tuition directly to enable students to take it. Funding limits it to England only at this stage but the growth in take-up is exceptional and extending.

The structure is in place for further growth, with 40% a realistic possibility for the proportion of mathematics A level students who should also do some form of further mathematics, about 20% doing AS, another 20% the full A level. These figures are realisable, with demand-pull from the university sector.

Further mathematics is not a luxury reserved for elite students. At A level it certainly presents an opportunity for the more mathematically-able students to be stretched and carried forward mathematically. At AS level it is a valuable qualification for any student wanting to do a mathematics-related subject at university. Anyone who wants to do engineering should be encouraged to take AS Further Mathematics. Such students are better prepared for their university studies and the transition is eased for universities.

The FMN has been working with university mathematics departments to assist in bridging the interface between school/college and university. Ways in which this can work are shown in statements from a growing list of universities (currently 28) who have been working with the FMN to encourage students to do further mathematics[18]. A wide range of universities is involved, not just the elite. Some talk about prospective mathematics undergraduates, others about engineering and physics. A good example is from Derby:

> **Mathematical and Computer Studies:** "Further Mathematics AS/A2 is not a requirement for entry onto our programmes, but if you have the opportunity to take further mathematics at AS or A level, we strongly recommend it. We find that

---

17 Since the Symposium took place, the 2007/8 figures have become available. These show further increases of 20.5% at AS-level and 15.5% at A level. This means that since the Further Mathematics Network began, Further Mathematics numbers have increased from 3,980 to 8,945, a 125% increase, at AS-level and from 5,720 to 9,091, a 59% increase, at A level.

18 What the universities say about further mathematics http://www.fmnetwork.org.uk/universities.php

> students who have taken these extra qualifications handle the transition from school or college to university very well. The University is prepared to be more flexible with students who have studied Further Mathematics but not met the standard offer."

Statements giving encouragement like that can make a real difference and it should become a point of honour for every university to join this list, not just for mathematics but for engineering and physics as well.

Schools want to help their students earn places at university and will do their best to provide what universities ask for. It is therefore within the power of universities to influence the take-up of mechanics, and its delivery through further mathematics, by encouraging students to take it. No longer does this need to be seen as unfair.

Universities can make their preferences clear in their offers; an example might be:

'Grade 'B' in A level Mathematics, including M1 and M2 **or** *grade 'C' in A level Mathematics plus grade 'D' in AS Further Mathematics, including M1 and M2 between them.'*

It is important to raise awareness and University College, London, has taken care to do this. When demanding A level Further Mathematics for students starting their Mathematics degree course, they contacted every London school to let them know of the support they could receive from FMN.

Schools range from those that can teach further mathematics but elect to do so only to their brightest A level students, to those who see no advantage in teaching it if universities do not require it, or those that see the advantages but have no teachers available to teach it. All can be helped and informed about the range of options that could benefit their students. There needs to be more and better communication between universities and schools about what universities want from new undergraduates and how schools and colleges can help provide it.

Findings and recommendations

Tuition through the Further Mathematics Network provides any student studying A level Mathematics at any school or college with the opportunity to take either M1 or M2 or both as part of AS or A level Further Mathematics.

The content of AS level Further Mathematics is accessible to any student capable of passing A level Mathematics.

AS Further Mathematics, taken alongside A level Mathematics in year 12 or 13, or across both years, is an ideal qualification for any student who intends to study for a degree in any area with a high mathematical content such as engineering or physics, as well as mathematics itself.

Increasingly universities are responding to the strong growth in popularity of both AS and A level Further Mathematics by encouraging students to take them, or making offers that include them.

Key recommendation 5

Access

Schools and colleges offering A level Mathematics should provide access to tuition in both Mechanics 1 and Mechanics 2, either as part of A level Mathematics or as part of AS/A level Further Mathematics.

Schools and colleges that cannot provide such tuition directly should collaborate with other local schools and colleges, or with external providers, to ensure their students can access tuition.

Recommendation 4.1 *Further Mathematics*
Schools and colleges should ensure that AS/A level Further Mathematics tuition is available to their students, and university departments should consider encouraging Further Mathematics in their prospectuses and offers.

At university

David Abrahams

The merger of the Victoria University of Manchester and UMIST in 2004 brought major change to the new University of Manchester, establishing a new School of Mathematics which in 2007 had an undergraduate intake of 450, the largest in the country.

As mechanics, and with it mathematical modelling and problem solving, was reduced in the school curriculum, Manchester's solution, for its large number of students, was a compulsory system with no choice at all for the first three semesters. To provide a continuum of mathematics from very pure to very applied this included mechanics and an introduction to applied mathematics. Recognizing what a good preparation further mathematics provides for an undergraduate programme, Manchester offered bursaries, doubled for three A's that included further mathematics. While there was no requirement, the incentive this gave led to a large increase in the number of students arriving with further mathematics. It meant a raised level of expectation of students and the chance to offer a challenging first year programme.

A key to success is strong schools and colleges liaison and Manchester has made financial investment in it. With two members of the Further Mathematics Network and three Teaching Fellows within the School of Mathematics this is an important focus. Individual staff members are involved with FMN, give master classes and summer schools, offering extra modules or stand-alone modules for school students to come and find out about what the School does. When the annual British Applied Mathematics Colloquium (previously the British Theoretical Mechanics Colloquium) was held in Manchester in Spring 2008, it included a day of talks and opportunities for school students to join in and understand that applied mathematics is a living subject. This should be rolled out as a public engagement exercise for the next three BAMCs.

There has been real enthusiasm and commitment from teachers in schools and more should be done by universities to encourage such liaison, to understand the issues and give help to deal with shortages of skills through CPD and other measures.

A range of action for university departments to take might include:

• Establish Schools & Colleges Liaison
  Assist in FMN, masterclasses, summer schools
  Improve dialogue regarding A level/university transition
  Provide support and CPD to teachers – in-service training in mechanics
  Careers information and support
• Develop more pedagogical material
• Emphasize the need for problem solving skills and mathematical modelling within the mathematics degree
• Increase take-up of FM by inducements/requirements
• Recommend M1+ in course documentation
• Mount events to stimulate school student interest in applied mathematics and mechanics.

The role of the Institute of Mathematics and its Applications (IMA)

The IMA, as a professional learned society for mathematics and its applications has about 4,500 members from commerce, finance, academia and the teaching profession. Although the number of school teachers is small, mathematics education is one of the primary activities of the IMA. It is a very strong focus for individual members, several represented at this symnposium, who are passionate about it. There is positive action already under way. Government is facing up to the agenda of a shortage of high quality mathematics teachers at all levels and the IMA, with other organisations is encouraging the accredited chartered status, CMathTeach, for specialist mathematics teachers. It will continue to support and endorse co-ordinating forums such as this, to lobby government and seek support from industry to improve the training shortage and improve the skills of those who are teaching mechanics.

Findings and recommendations

More and better communication is needed between universities and schools. Some universities have created effective links with local schools and business or through such bodies as the IMA, the network of STEMNET ambassadors and local mathematics advisers. 'Mechanics Missionaries', masterclasses, courses and events can stimulate interest in modelling and problem solving and at the same time provide a useful means of two-way information sharing.

Universities include increasing numbers of overseas staff unfamiliar with students' school background and experience and with different expectations of the place of mechanics not just in applied but in pure mathematics courses.

Recommendation 4.2: *school/college/university links* Links should be strengthened between schools and colleges and university engineering, physics and mathematics departments emphasizing the importance of mathematics in these disciplines.

Recommendation 4.3: *school/college/university training links* Co-operation should be explored between universities and schools and colleges over training opportunities for university staff and for school and college teachers.

The Engineering Diploma

Fred Maillardet

Many of the issues raised throughout the Symposium relate closely to the concerns of the Engineering Professors Council (EPC), prompting its setting up of the Mathematics Working Group in 2001 and its contribution to the development of Level 3 of the Engineering Diploma.[19]

When plans were first made for the Diploma many EPC members were delighted to see the intention to bridge the academic-vocational divide. The diplomas were designed to lead to apprenticeships, work or further study, the aspect which would be the focus of EPC, whose initial concerns would be for the mathematical content, teachers' ability to deliver it and the real level of industrial involvement.

Mathematical Techniques and Applications for Engineers was to form part of Principal Learning, a compulsory element of the Engineering Diploma. Initially it offered four topics to be delivered in 60 guided learning hours, quite inadequate as preparation for further study. So to address this the Mathematics Working Group set up a Task Group, increasing its membership to include the National Centre for Excellence in the Teaching of Mathematics, Mathematics in Education and Industry, the Engineering Development Partnership and helpful support from the Qualifications and Curriculum Authority. With their permission, Loughborough University's excellent foundation course designed for students without traditional A levels, formed the basis of a more extensive unit, adding a further 180 learning hours for students wanting to take a degree. With content similar to A level, Loughborough students' performance has often exceeded those with traditional A level qualifications.

The key element here, though, is the applications orientation. If mathematics is the language of engineering it must be seen in the context of real engineering applications and Newtonian mechanics features in many of these. To fit each topic in the unit, exemplars are being developed with relevant industrial companies, for example JCB, Rolls-Royce, Thales and Npower, using the work they are doing to demonstrate and stimulate young people's interest and excitement. Solving real problems gives a sense of achievement and enthusiasm, can lead to understanding and defeat the 'can't do' attitude.

The awarding body OCR has accreditation for providing the award from September 2008[20] and the structure of the exam developed with OCR explores students' ability to apply their knowledge. The first part consists of traditional mathematics questions, but the second part is released a week or two in advance, setting a context and requiring answers to questions to show how students can actually apply their understanding. Students' reaction to a small-scale trial showed they found it a challenge, were tripped up on occasion by technical language but appreciated that it was real engineering.

Experience in preparing the Diploma shows that real engineering applications can make mathematics attractive and understandable to a much wider audience, but more support for teachers is needed for it to succeed. The Diploma offers a new approach to gaining mathematical modelling and problem solving skills. It gives an opportunity for genuinely widening participation and awakening an interest in mathematics in people who have not looked at it this way before.

19 http://www.engineeringdiploma.com/

20 http://www.ocr.org.uk/qualifications/diplomas/engineering_level3/

Findings and recommendations

Loughborough University made available its mathematics foundation course for engineering students as a basis for the mathematics unit within the Level 3 Engineering Diploma with its options for leading into further study. The strengths of introducing mechanics concepts connected directly to their real world applications have been evident in the pre-launch trials.

Mathematics is a key element of the Level 3 Engineering Diploma and the application of mathematics to real problems is crucial to engineering.

Whilst students find 'real world' problems challenging, they are also strongly motivated by them.

Recommendation 4.4: *teaching in context* Wherever possible, mechanics should be taught in the context of real world problems.

Recommendation 4.5: *exemplars* The exemplars that are being produced to support the Level 3 Engineering Diploma should be made available to all teachers and students of mechanics at school, college and university.

5 Discussion groups and Resources

See Appendix 2 for summary notes from the discussion groups, See Appendix 3 for a list of demonstrations and resources

Issues raised in the presentations stimulated immediate questions and were explored further in later breakout group discussions. Key findings and recommendations given on each topic in this report reflect not only the topic presentation itself but the points raised and agreed in those discussions.

In addition, the two questions the groups discussed in detail raised further issues which led to the findings and recommendations below.

Question 1: What constitutes an adequate preparation in Newtonian mechanics for students entering Higher Education in Engineering, Physics and Applied Mathematics?

An adequate preparation, it was agreed, needs to start early. Children should receive progressive exposure to open-ended problems so that by Key Stage 3 problem solving should be part of the mathematics curriculum. It was felt there were two main obstacles to achieving this which must be overcome. Firstly, mechanisms for assessment, particularly the introduction of electronic marking, lacked the flexibility to assess problem solving and secondly, many teachers lacked the training and practical skills to introduce it.

As students approach the transition to university, the question was considered whether mechanics really is the best vehicle or whether there might be other routes to modelling and problem-solving. It had to be borne in mind that many students are undecided what course they might eventually take. It was felt that teachers approved and could work with the current structure of A and AS level mathematics options.

There was, however, general concern about the lack of mathematics in physics. There may be limited study time available, there may be qualified teacher shortages, but the need to develop mathematical argument over several steps needed to be reintroduced into the A level Physics curriculum.

Findings and recommendations

The Mathematics National Curriculum has recently been revised to encourage a more flexible approach to teaching and learning at Key Stages 3 and 4.

There is concern in schools, colleges and universities at the loss of mathematical content from the physics A level curriculum.

Recommendation 5.1: *modelling and problem solving at KS3 and 4* Mathematical modelling should be introduced at an early stage, tailored to the development and understanding of different age groups.

Recommendation 5.2: *mathematics in A level physics* The mathematical content of A level Physics should be strengthened, restoring it to pre-Curriculum 2000 levels.

Question 2: How can teachers and students be supported in studying Newtonian mechanics, both at school/college and university levels?

Many of the ways in which teachers and students can be supported have been suggested earlier in this report. Further practical ideas were put forward and discussed:

Diagnostics

Students' A level or other qualifications can give only the broadest indication of their mathematical understanding, so diagnostic tests on entry to a physics or engineering course can help students recognize their strengths and weaknesses and advise staff about their general skills.

Resources

There are good resources on the market for schools, to support the curriculum and for classroom whiteboard use, but areas were identified where resources are not yet available, are needed and could make a significant difference.

There is a need for innovative mathematics resources for use at the school-university interface, to support CPD, to encourage interest and enthusiasm and to support individual learning for students. Authoritative, confidence-building materials are needed to supplement staff training, provide a benchmark for lecturers from overseas and give opportunities for practice and revision to students at the transition. Prepared by a group equally representative of schools and universities, for use in both schools and universities, such resources would provide an immediate and effective means towards bridging the gap at the school/university interface and complement other school/ university links being developed. An immediate, practical solution would be to extend the freely available ***Math****centre* website's ***Math****tutor* resource to include mechanics topics.

Resources might include digital video to promote:

(i) mechanics in action,

(ii) the importance of mathematical modelling and problem solving in a range of professions and industries,

(iii) video mechanics tutorials.

Other resources might include:

(i) mechanics kits,

(ii) mechanics texts,

(iii) interactive mechanics investigations.

Findings and recommendations

There is a need for innovative mathematics resources for use at the school-university interface, to support CPD, to encourage interest and enthusiasm and to support individual learning for students.

SCENTA's *A Rough Guide to Designing your Future* and online careers advice is available but guidance is needed on the range of STEM A level and Diploma options at year 12.

Key recommendation 6

Resources

Funding should be made available to create innovative and inspirational mechanics resources and integrate them into the teaching and learning of mechanics on both sides of the school/college – university interface.

The emphasis of these resources should be to help develop students' modelling and problem-solving skills.

Recommendation 5.3: *guide to A level and Diploma choices* A 'Rough Guide' to STEM subjects for students entering year 12 should be produced. It should make clear the opportunities arising from A level and Diploma choices.

The work of Loughborough and Coventry Universities, through **sigma** (the centre for excellence in university-wide mathematics and statistics support) in developing and enhancing mathematics support centres has encouraged growth of such facilities across the sector. Increasingly such centres are offering limited mechanics support in addition to the mathematics support traditionally provided. Many sigma resources are freely available on-line and are promoted by the Higher Education Academy Mathematics, Statistics & Operational Research Subject Centre but there are currently few resources to support students with mechanics.

Recommendation 5.4: *investigation of universities' mechanics support* The HEA Maths, Stats & OR, Engineering, Materials and Physical Sciences Subject Centres should be commissioned to investigate mechanics support across the sector to highlight the most effective initiatives for wider take-up and development and to avoid duplication of effort.

Summary of findings and recommendations

1 The importance of Newton's mechanics

Newtonian mechanics has a vital role to play in the education of potential mathematicians, physical scientists and engineers; it

- introduces important concepts,
- provides ample scope for applying mathematics to solving real physical problems,
- develops students' mathematical modelling and problem solving skills,
- improves students' fluency in algebra and calculus.

Mathematical modelling and problem solving skills are generic skills which, once mastered, prepare students for problem solving in all branches of science and engineering. These skills are also used by creative thinkers in industry and in business and commerce.

The ability of teachers to inspire young people with their own enthusiasm for the subject can play a vital role in encouraging take-up of the subject, by staff as well as students.

Key recommendation 1

Careers

Universities and employers should build upon links with schools and colleges to emphasize that degrees which develop mathematical modelling and problem solving skills, based on a foundation of Newtonian mechanics, can lead to stimulating and rewarding careers.

2 The mechanics curriculum in decline

Changes to A level Mathematics 1960-2008

Currently 30-40% of schools and colleges enter their students for at most one unit of mechanics. As a result, many students arrive at university to study physics, engineering and mathematics having studied at most one unit of mechanics.

The reduction in the amount of Newtonian mechanics studied by many students as part of A level Mathematics has resulted in a decline in students' modelling and problem solving skills.

Teachers are generally happy with the current structure of AS/A level Mathematics and Further Mathematics.

Recommendation 2.1 *AS/A level Mathematics and Further Mathematics* Short term changes to the current structure of AS/A level Mathematics and Further Mathematics before the 2013 review should be avoided.

Recommendation 2.2 *AS/A level Mathematics* A level Mathematics should remain at 6 modules to avoid the risk of reducing the amount of mechanics available to students and fragmenting the applied mathematics curriculum.

Key recommendation 2

Student information

Schools and colleges should ensure that students intending to study physics or engineering in Higher Education know that:

(i) the content of mechanics units within AS/A level Mathematics provides important background knowledge for such university courses; and

(ii) studying mechanics is an excellent way to develop mathematical modelling and problem solving skills.

Teacher training
There is a shortage of mathematics teachers with the skills to teach mechanics as part of A level Mathematics or Physics. CPD in mechanics is not readily available and there are difficulties for schools in finding cover for teachers to attend.

Key recommendation 3

Teacher training

Courses and resources in mechanics should be provided:

(i) within initial teacher training for all secondary mathematics and physics teachers; and

(ii) for continuing professional development of practising mathematics and physics teachers.

3 The impact on HE of A level mechanics decline

Measuring the mechanics problem in UK universities
Universities report that many students currently admitted onto degree courses in physics, engineering and mathematics, even those with top grades in A level Mathematics, have an inadequate preparation in mathematical modelling and problem solving skills and a poor knowledge of mechanics.

Studying just one unit of mechanics, M1, at A level provides students with a basic introduction, but does not constitute an adequate preparation for physics or engineering in Higher Education.

Most university departments of physics, engineering and mathematics will prefer and some will expect prospective students to have taken two units of mechanics (M1 and M2) as part of A level Mathematics or AS/A level Further Mathematics.

Most university departments of physics and engineering admit some students who are well prepared in mechanics and others who are ill-prepared. The latter may be seriously disadvantaged unless special provision is made for them. Diagnostic tests can signal the importance of mechanics for the degree course, alert staff and students to students' strengths and weaknesses, but must be well-designed and do require follow-up.

Key recommendation 4

Universities' expectations

University departments of physics, engineering and mathematics should specify how much mechanics they would like prospective students to have studied.

Recommendation 3.1 *Universities' information and update* University departments of physics and engineering should:

(i) monitor changes in A level Mathematics and Further Mathematics and other relevant pre-university qualifications;

(ii) gain an understanding of incoming students' prior knowledge of mechanics; and

(iii) review teaching methods and styles to accommodate students with varying levels of mechanics knowledge.

Mind the gap! a transition problem across Europe
Response from the EUPEN group of 160 European universities reveals a widening gap in students' knowledge of mechanics at the transition to university study in physics, and that this gap is most severe in the UK. Pending publication of its report, it is anticipated that EUPEN recommendations will include improving communication between schools and universities, particularly over teacher training and collaboration over development of pedagogical materials. It will also recommend that steps should be taken to enhance the links between physics and mathematics in the school curriculum.

Note: The EUPEN recommendations are in accord with this symposium's recommendations 4.2, 4.3 and 5.2.

4 Enhancing the take-up of Newtonian mechanics at ages 16-19

at school – the Further Mathematics Network

The Further Mathematics Network provides any student studying A level Mathematics at any school with the opportunity to take both M1 and M2 as part of A level Mathematics or to take M1 as part of A level Mathematics and M2 as part of AS or A level Further Mathematics.

Both AS and A level Further Mathematics are growing in popularity and increasingly university departments are encouraging students to take them, or are making offers that include them.

Recommendation 4.1 *Further Mathematics* Schools and colleges should ensure that AS/A level Further Mathematics tuition is available to their students, and university departments should consider encouraging Further Mathematics in their prospectuses and offers.

Key recommendation 5

Access

Schools and colleges offering A level Mathematics should provide access to tuition in both Mechanics 1 and Mechanics 2, either as part of A level Mathematics or as part of AS/A level Further Mathematics.

Schools and colleges that cannot provide such tuition directly should collaborate with other local schools and colleges, or with external providers, to ensure their students can access tuition.

at university

More and better communication is needed between universities and schools. Some universities have created effective links with local schools and business or through such bodies as the IMA, the network of STEMNET ambassadors and local mathematics advisers. 'Mechanics Missionaries', masterclasses, courses and events can stimulate interest in modelling and problem solving and at the same time provide a useful means of two-way information sharing.

Universities include increasing numbers of overseas staff unfamiliar with students' school background and experience and with different expectations of the place of mechanics not just in applied but in pure mathematics courses.

Recommendation 4.2: *school/college/university links* Links should be strengthened between schools and colleges and university engineering, physics and mathematics departments emphasizing the importance of mathematics in these disciplines.

Recommendation 4.3: *school/college/university training links* Co-operation should be explored between universities and schools and colleges over training opportunities for university staff and for school and college teachers.

the Engineering Diploma

Loughborough University made available its mathematics foundation course for engineering students as a basis for the mathematics unit within the Level 3 Engineering Diploma. The strengths of introducing mechanics concepts connected directly to their real world applications have been evident in the pre-launch trials.

Recommendation 4.4: *teaching in context* Wherever possible, mechanics should be taught in the context of real world problems.

Recommendation 4.5: *exemplars* The exemplars that are being produced to support the Level 3 Engineering Diploma should be made available to all teachers and students of mechanics at school, college and university.

5 Discussion Groups and Resources

school curriculum

The Mathematics National Curriculum has recently been revised to encourage a more flexible approach to teaching and learning at Key Stages 3 and 4.

There is concern in schools, colleges and universities at the loss of mathematical content from the physics A level curriculum.

Recommendation 5.1: *modelling and problem solving at KS3 and 4* Mathematical modelling should be introduced at an early stage, tailored to the development and understanding of different age groups.

Recommendation 5.2: *mathematics in A level physics* The mathematical content of A level Physics should be strengthened, restoring it to pre-Curriculum 2000 levels.

teaching and learning resources

There is a need for innovative mathematics resources for use at the school-university interface, to support CPD, to encourage interest and enthusiasm and to support individual learning for students.

SCENTA's *A Rough Guide to Designing your Future* and online careers advice is available but guidance is needed on the range of STEM A level and and Diploma options at year 12.

Recommendation 5.3: *guide to A level and Diploma choices* A 'Rough Guide' to STEM subjects for students entering year 12 should be produced. It should make clear the opportunities arising from A level and Diploma choices.

The work of Loughborough and Coventry Universities, through **sigma** (the centre for excellence in university-wide mathematics and statistics support) in developing and enhancing mathematics support centres has encouraged growth of such facilities across the sector. Increasingly such centres are offering limited mechanics support in addition to the mathematics support traditionally provided. Many sigma resources are freely available on-line and are promoted by the HEA MSOR Subject Centre but there are currently few resources to support students with mechanics.

Recommendation 5.4: *investigation of universities' mechanics support* The HEA MSOR, Engineering, Materials and Physical Sciences Subject Centres should be commissioned to investigate mechanics support across the sector to highlight the most effective initiatives for wider take-up and development and to avoid duplication of effort.

Key recommendation 6

Resources

Funding should be made available to create innovative and inspirational mechanics resources and integrate them into the teaching and learning of mechanics on both sides of the school/college – university interface.

The emphasis of these resources should be to help develop students' modelling and problem-solving skills.

Appendix 1

The structure of AS/A level Mathematics and Further Mathematics 2008

AS and A level Mathematics

There are four core 'pure' mathematics units. Core 1 (C1) and Core 2 (C2) are at AS standard; Core 3 (C3) and Core 4 (C4) are at A2 standard and depend on C1 and C2.

There are three 'applied' mathematics strands, mechanics (M), statistics (S) and decision (D). M1, S1 and D1 are AS units. M2, S2 and D2 are A2 standard and depend on the corresponding AS units, so M2 depends on M1, etc.

AS Mathematics is made up of three equally weighted units: The two AS 'pure' mathematics units, C1 and C2 are compulsory; the other is an 'applied' mathematics unit and can be chosen from M1, S1 or D1.

A2 Mathematics follows on from AS Mathematics and is also made up of three equally weighted units: The two A2 'pure' mathematics units, C3 and C4, are compulsory; the other is an 'applied' mathematics unit and can be chosen in the following ways:

- If a student has done M1 in AS Mathematics, they can choose from M2, S1 and D1.
- If a student has done S1 in AS Mathematics, they can choose from S2, M1 and D1.
- If a student has done D1 in AS Mathematics, they can choose from D2, M1 and D1.

This means ***A level Mathematics*** (AS + A2) can comprise C1, C2, C3 and C4 plus either:

M1 and M2; M1 and S1; M1 and D1; S1 and S2; S1 and D1; D1 and D2. Most students take M1 and S1, so one sixth of their A level is mechanics. For students who take M1 and M2, one third of their A level is mechanics. Students who take S1 and D1, S1 and S2 or D1 and D2 do no mechanics at all.

AS and A level Further Mathematics

AS/A level Further Mathematics must be studied alongside or after AS/A level Mathematics.

The fine detail depends on which examination awarding body's specification is used, but the general structure is as follows.

There is one AS Further Pure unit, FP1, and up to three A2 Further Pure units, FP2, FP3 and FP4. The A2 Further Pure units are dependent on FP1.

There are also further applied mathematics units in the mechanics and statistics strands, M3, M4, S3 and S4.

AS Further Mathematics is made up from three equally weighted units: One AS 'pure' mathematics unit, Further Pure 1 (FP1) is compulsory; the other two units can be chosen from M1, M2, M3, M4, S1, S2, S3, S4, D1, D2, FP2, FP3 or FP4. No applied unit that is used in a student's AS or A level Mathematics may also be used in that student's AS or A level Further Mathematics.

A useful unit combination for prospective engineering or physics undergraduates for AS Further Mathematics could be FP1, M1, M2 or FP1, M2, M3 if M1 was part of their AS/A level Mathematics.

A2 Further Mathematics is made up from three equally weighted units.

One 'pure' A2 unit is compulsory, but there can be a choice of which one.

The other two units can be chosen from any of the A2 units not used in AS/A level Mathematics or AS Further Mathematics.

A useful unit combination for a prospective engineering or physics undergraduate for A level Further Mathematics (AS + A2) could be FP1, FP2, M1, M2, M3, S2 or FP1, FP2, FP3, M2, M3, M4.

Appendix 2

Discussion Groups Summary Notes

The symposium participants were divided into 4 breakout groups, which met in separate rooms for two sessions of roughly an hour each before and after the lunch break on Day 2.

First Session

A. Groups 1 and 2 considered the following question:

What constitutes an adequate preparation in Newtonian mechanics for students entering Higher Education in Engineering, Physics and Applied Mathematics?

Group 1 summarized their deliberations as follows:

- Keep the diversity in the current structure of A level Mathematics and Further Mathematics, which serve many different needs and constituencies.
- Teachers like the present structure, feel it works, and would welcome a period of stability.
- In Science, Technology, Engineering and Mathematics (STEM) students should be encouraged to take M1, and if possible, M2. Stronger university departments (the selectors rather than recruiters) should insist on M1, and even M2, for admission to STEM degree courses, especially now that Further Mathematics is universally available again.
- Modelling should be appropriately introduced as early as possible in the National Curriculum.
- A "Rough Guide to STEM subjects" for students entering Year 12 should be written and approved by all main stakeholders. It should make consequences of A level choices clear.
- Mechanics (especially with M2) is the strongest vehicle for learning modelling even though Stats and Decision Mathematics also offer valuable modelling experience.
- Communication between schools and universities needs to be improved so that both sides of the divide understand each others' needs and limitations. A few universities successfully put resources into fostering good relations with local schools.
- Please put the mathematics back into Physics, at all levels, but especially at A level and beyond.

Group 2 devoted the first half of their session to the question whether Mechanics is the best vehicle for learning mathematical modelling and problem solving. The following points were made:

- Modelling in the Stats and Decision Mathematics modules is more limited than in Mechanics, which deals more broadly with the outside world.
- Students can do well in S1 and D1 without developing extensive modelling/problem skills. Teachers should be encouraged to emphasize those parts of Statistics and Decision Mathematics that develop these skills.
- Modelling in Physics has more varied contexts than straight Mechanics. Mechanics is wide enough and yet has a limited number of principles that can be learnt in the study time available.
- A fairly small percentage of students taking A level Mathematics go on to STEM degree courses; a higher proportion do Business, Management, Economics and Psychology degrees, which require Statistics. Schools often do not know their students' final destinations in Year 12. There are yet other students who need only a broad cultural understanding of the value of modelling and problem solving (eg future decision-makers in educational policy and funding).
- Teaching modelling in a tangible context develops students' confidence, but too many different contexts can get in the way of understanding. The process of generalising and abstracting (where one bit of mathematics is the key to solving many different problems) is a high level skill and does not come easily to most A level students.

The remaining time of Group 2's session was taken up with practical and policy issues:

- To improve their points score in the league tables, many schools will choose to offer only M1 and S1 (and perhaps D1).
- While it is unrealistic to call for compulsory M1 in A level Mathematics, it could reasonably become core

for students also taking AS Further Mathematics, which can now be delivered through the Further Mathematics Network.

- Students studying STEM subjects at university struggle if they arrive without any Mechanics. If universities want a more homogeneous intake, they should make M1 an admissions requirement.
- Over the next 12 months or so, decisions will be made about the shape of A level Mathematics from 2011 on; a reduction from 6 to 4 modules is being considered. This is an opportunity to push for a component (20%) of compulsory Mechanics. But there is a risk that Mechanics in the new core could turn students off.
- There is a push from Government for "*personalisation*" of students' individual curriculums. At the same time, under a new funding model, schools are to be rewarded for offering a broader choice of A level subjects with a scaling factor (of 1.2 or 1.4) applied to their funding.
- Schools are paid only for 4.25 A level modules per student; therefore students taking an additional Mechanics module in Year 13 would bring in extra funding.

B. Groups 3 and 4 considered a different question:

How can teachers and students be supported in studying Newtonian mechanics, both at school/college and university levels?

Group 3 began with a discussion of the value of online resources and diagnostic testing, and went on to look at ways to improve the take-up:

- Solutions to the "*Mathematics Problem*" now apply to a fairly homogeneous cohort of A level students, in that the students have all done the modules C1 – C4. The "*Mechanics Problem*" is different because some students have done no Mechanics modules while others may have done one or two, and exceptionally three or four.
- The *Mathcentre* website is a widely-used resource, relatively low-tech and inexpensive to create and run; it is supported with the *Mathtutor* video tutorials and exercises. Could it be used to distribute additional materials to support students (secondary and tertiary) learning Mechanics?
- Diagnostic tests in Mechanics for first-year students in STEM subjects have many advantages:

  they signal the importance of Mathematics and Mechanics in the degree course;

  students "hit the ground running" and can be warned of the imminent test over the summer before they start;

  they tell students about their strengths and weaknesses; and

  they tell staff about the general level of skills.
- Well-designed banks of Applied Mathematics questions, centrally accessible, customizable by staff, and supplied with generous feedback could be used by schools, colleges and universities in a variety of modes, including diagnostic testing.
- An effective and inexpensive way to deliver online questions with feedback is offered by software such as *Camtasia*, which allows a teacher to create hand-written solutions unfolding in a web browser with a voice over, with virtually no training or special expertise. Students submit a question and are then offered the chance to watch someone writing out the solution with a commentary on their thought processes.
- Students who have done mechanics modules cannot be assumed always to have the expected competences, but they are 'more teachable' in these areas.
- Exemplars of mechanics in action (short videos of applications in industry and elsewhere) such as those being prepared for the Engineering Diploma would be a useful asset, especially if accompanied by a teachers' guide to show how they can be most effectively used. There are good tools for creating digital resources (eg OpenMark at the OU, Geogebra) but they require expertise and investment to be put to effective use.

  There is a shortage of new teachers who are confident to deliver Mechanics modules. Short CPD courses will not remedy this. Longer-term subject-knowledge training (Government policy is to move teaching towards a 'Masters degree' profession), or possibly teachers trained in two areas like Physics and Mathematics are possible ways to increase supply.
- Although some university STEM departments with recruiting problems are under pressure to be less prescriptive in their admission policy, this should be resisted. The numbers studying A level Mathematics are increasing and by encouraging students to take Mechanics modules, parents will press for it and teachers will have ammunition for persuading their senior management to provide it. Some universities have offered financial incentives to applicants with desirable A levels such as Further Mathematics.
- Among the reasons why schools don't offer M1 and M2 are:

  shortage of teachers with Mechanics experience/competence;

schools' concern about league tables; and

small classes.

Universities can influence this situation positively through their admissions policies. With AS Further Mathematics now accessible to all students in England through the Further Mathematics Network, it is comfortably possible to take four applied modules, including M1 and M2 if appropriate.

Group 4 started with the question:

"Should students entering Physics and Engineering degrees take a diagnostic test in Mathematics/Mechanics?"

With important provisos, the consensus was for an affirmative answer. The supporting reasons included:

- the increasingly diverse backgrounds and range of qualifications in student intakes makes very hard to compare levels of preparedness;
- knowing students' marks in individual A level modules is useful but not sufficiently reliable to make comparisons from year to year and with other qualifications;
- starting from scratch, with a module that assumes students know no mechanics when they arrive at university, is a common and easily-implemented solution but risks boring the well-prepared students and losing the poorly-prepared ones. It is not ideal for large inhomogeneous classes;
- well-designed diagnostic tests can identify students at risk as well as students who are ready to move on to more challenging material.

The provisos included:

- Diagnostic tests are useless without follow-up that:

  informs lecturers of their students' knowledge and understanding; and identifies students' needs and tries to meet them.
- Trained university teachers and appropriate resources should be concentrated on those that most need them; weaker students benefit from small-group special teaching early on.
- Modelling and problem-solving abilities take time to develop and are best begun well before university. Short crash courses at university are not a substitute.

In the second half of the session **Group 4** moved on to the question of how best to support teachers and students in schools/colleges. These points were made:

- There was some strong support for making M1 and S1 compulsory in A level (*downside*: might put off some potential takers of A level Mathematics).
- The shortage of teachers with a Mathematics or Physics degree is a serious problem. On the other hand, requiring more Mathematics in the Physics A level might lead to the demise of Physics A level teaching in some schools[21]
- A good alternative would be for STEM subjects in universities to ask for M1 in their admissions requirements and guidance. Such a move might be coordinated through the Conference of Heads of Departments of Mathematical Siences (HoDoMS) or through the HEA subject centres for STEM subjects.
- More CPD in Mechanics for teachers would be a boon (preferably carried out over a longer period via video conferencing with teachers in schools and hands-on workshops organised, for instance, by the Further Mathematics Network). A recommendation should be that the FMN be funded to provide CPD for teachers, especially in Mechanics. The infrastructure and expertise exists and the name is trusted and respected and it already has good links with schools and consultants, so it should be used.

Second Session

After lunch, **all groups** addressed the question:

What recommendations can we make to improve new undergraduates' mathematical modelling and problem-solving skills?

Group 1 summarised their discussion points and recommendations as follows:

When students come to university they should be familiar with the idea of problem solving.

The current curriculum

- The structure of questions is generally that they can be solved in one step; there is rarely a need to develop an argument over several steps. The current system of assessment is getting in the way of developing problem solving skills; this is likely to get worse with the introduction of electronic marking which will not have the flexibility for assessing problem solving.
- There are practical difficulties with introducing modelling in schools. Can we, as a community, write a paper for influential people making recommendations to see if we can address these problems?

[21] The week after the symposium saw the announcement of the £30M Project ENTHUSE , focusing on science CPD at the National Science Learning Centre in York.

Students

- Developing problem solving skills should be part of all students' mathematical learning. This should occur at a much younger age, say Key Stage 3.
- Students should be given exposure to mathematical modelling and solving open-ended problems.

Teachers

- Mathematics teachers are under pressure to get students through the exams. We cannot expect teachers to focus on developing modelling and problem solving when they don't have the time and resources to do it. Resources for training and for use in the classroom must be made available.

Universities

- Universities must try to stimulate problem solving through school/college liaison. Some universities already engage with local schools and it is recommended that all universities should do so by running mathematical modelling and problem soloing days for local school students. It is understood that this may create some problems if schools do not engage or are unwilling to release students from timetable for a day. In order to maximise impact there needs to be a change of culture.
- Some universities already run modelling and problem solving courses, these need to be more widespread. Again assessment can be a problem; these types pf courses are hard to design and assess.

Group 2 summarised their discussion points and recommendations as follows:

Students

- Must be given the chance to acquire modelling skills by practising, ideally at school before coming to university.
- There is a difficulty with introducing modelling at A level because there is no credit given to it. Exam questions are set and assessed by the level of difficulty of the mathematics, and credit is not given for the level of difficulty of the process needed to get to the mathematics. Hence there is no scope in the current A level for introducing and assessing modelling skills.
- The group could recommend that at least some questions on A level papers do not lead students through step by step but go back to the type of problem where students need to think their way through the problem.

Teachers

- There are many schools where teachers of mathematics and physics are not confident and sometimes not excited about problem solving. They must be helped to develop their skills and confidence.
- A strong recommendation must be that resources are made available for training existing teachers. The group understands that there is pressure on schools, particularly in areas where there is a shortage of teachers but teachers must be given the opportunity to improve their knowledge.

Universities

- Universities must spell out exactly what they want students to have.
- If students lack modelling skills then universities will need to provide introductory courses. These can be done as bridging courses at the very beginning of a degree, or sometime in the first year when the assessment does not (usually) count towards the final degree.

Careers Advice

- Universities and employers must inform teachers and students about the need to study mechanics.
- Universities and employers must inform teachers and students that a mathematics degree can lead to interesting and lucrative jobs
- Schools should advise students wishing to go on to study mathematics, physics and engineering that they must study at least M1, preferably more mechanics modules
- If schools cannot offer mechanics modules, they should use the FMN to deliver these modules to students who need them.
- Design posters showing interesting and simple modelling activities that lead to interesting outcomes that are sent to all schools.

Group 3 summarised their discussion points and recommendations as follows:

- The group agreed with the previous groups. The group discussion focused on modelling in schools and they had some more philosophical discussion about what mathematical modelling actually is and whether the ability to learn and do new mathematics is necessarily the same as being able to apply mathematics and do mathematical modelling.
- Should mathematical modelling in schools be confined to A level?
- Is mathematical modelling necessarily addressed through coursework? The assessment of coursework has proved problematic and thus it is unlikely to be included in A level.
- Is it better not to teach modelling at all in schools but to focus on becoming fluent with the mathematics and leave teaching modelling until students get to university.

- There was discussion of the use of software in mathematical modelling and whether this would be suitable/available in schools.

Group 4 summarised their discussion points and recommendations as follows

Both problem solving and mathematical skills are improved and enhanced by studying mechanics which is why it is such an important area to study.

The current curriculum

- Coursework is no longer part of the curriculum as it takes time that needs to be spent covering the syllabus.
- Open ended questions are not the way that school assessment is developing. The current trend in school assessment reduces the likelihood of problem solving and open ended tasks being incorporated into the curriculum.

Students

- We should encourage schools to use enhancement activities that include mathematical modelling but are not assessed.
- Design and technology in schools is one area where there is an opportunity for open-ended problem solving – could projects be done in conjunction with mathematics?

Teachers

- A strong emphasis should be put on supporting CPD for teachers. There are good resources and training available and teachers should be encouraged to access this.
- Sir Peter Williams' initiative to put a specialist mathematics teacher in every primary school is one which the mathematics community should engage with. We can ensure that these teachers have some exposure to the likely future demands on any pupils who have some mathematical flair.

Universities

- The group emphasized that universities need to specify the need for mechanics modules above M1 when they would like them.
- One obvious solution is for all students who do STEM subjects to have at least an AS level in Further Mathematics. If universities could agree on this it would become the norm.
- Universities should engage with and seek support from the FMN and their local Further Mathematics Network Centre.
- Universities should make contact with the local Mathematics advisor/ consultant. Through them they can make better links with schools and parents
- It was suggested that universities hold an annual meeting for parents of school students in their area to explain the consequences of the A level choices.
- Ensure that universities are aware of the STEMNET science and engineering ambassadors who will go into schools to talk about STEM at work and could also take the message about appropriate subject choices in the sixth form.
- Understand the value to schools of masterclasses at universities to stimulate problem solving. These can inspire students.
- Subject Centres and the Higher Education Academy could do some research into what support is currently available to first year undergraduates with weak mathematics skills.

Appendix 3

Demonstrations and Resources

A range of demonstrations to show some of the activities, initiatives and resources currently being used to support the teaching of mechanics in schools, colleges and universities contributed to the discussions on support for teachers and students.

MathTutor & MechanicsTutor Prototypes
www.mathtutor.ac.uk & www.mathcentre.ac.uk
The **math**tutor resource provides material that covers most of the mathematics needed for the transition from school to university. The eighty topics are presented using videos, with interactive diagnostic tests and exercises; printed summaries of the videos and exercises are also available.

MechanicsTutor: Circular Motion
www.symplekta.co.uk/MTpreview
This prototype resource contains a set of nine demonstration videos on various aspects of motion in a horizontal circle, together with three additional problem-solving videos on linear motion and a first attempt at an extended interactive exercise featuring the conical pendulum.

IPMathematics
www.fable.co.uk/ipmathematics.htm
IPMathematics is an intuitive and powerful mechanics modelling program that allows teachers and students to rapidly build, model and measure mechanical systems. It comes with a library of example models for the mechanics modules and is already used by mathematics departments in 500+ UK schools and colleges. Also on display is a new mathematics mechanics M1 project that is currently in development. This interactive online resource uses compelling 3D game-like models to explain mechanics M1 kinematics concepts.

MEI online Resources for Mathematics
www.resources.mei.org.uk
An extensive website of resources to support teachers and students of A level Mathematics. The resources include text-based pages, interactive resources and multiple-choice assessments. The resources contain separate materials to support mechanics in all 4 main English A level specifications.

'The Magic of Mechanics'
A series of experiments and investigations that have been designed and developed to support enrichment programmes for 'Young Gifted and Talented' students, from Primary to Post 16 level, at Loughborough University.

Professional Development for Teachers
MEI and the Further Mathematics Network offer a variety of professional development courses for teachers to support them in teaching mechanics.

Participants

Sir Peter Williams, The Royal Society (Chair)
Professor David Abrahams, University of Manchester (Co-chair)
Dr John Morton, Engineering and Technology Board (Co-chair)
Professor Tim Pedley, University of Cambridge (Co-chair)
Mr Andrew Ramsay, Engineering Council, UK (Co-chair)
Dr Mohamad Askari, Kingston University
Dr Mike Barry, University of Bristol
Dr Rob Best, London South Bank University
Mrs Miggy Biller, York College
Dr Marion Birch, University of Manchester
Mr Andrew Blessley, Clothworkers' Company
Dr Rod Bond, Further Mathematics Network
Dr Brian Brooks, Millfield School,
Mr Richard Browne, Mathematics in Education and Industry
Dr Simon Carson, Norton College
Dr Neil Challis, Sheffield-Hallam University
Professor Dick Clements, University of Bristol
Professor Brian Cowan, Royal Holloway, University of London
Dr Lionel Elliott, University of Leeds,
Dr Martin Greenhow, Brunel University
Dr Martin Harrison, Loughborough University
Dr Stephen Hibberd, University of Nottingham
Mr David Holland, A level Mathematics Examiner
Professor Derek Ingham, University of Leeds,
Miss Brenda Jennison, University of Cambridge,
Professor Gareth Jones, Imperial College, London,
Dr Joe Kyle, MSOR Subject Centre
Dr David Leppinen, University of Birmingham
Professor Angus Mackinnon, Imperial College, London
Professor Fred Maillardet, Engineering Professors' Council
Mr Gerard McBreen, Fable Multimedia
Dr Liz Meenan, University of Leeds
Mr Phil Moxon, The Further Mathematics Network
Dr Les Mustoe, Loughborough University
Ms Dawn Ohlson, Thales UK plc
Dr Geoff Parks, University of Cambridge
Mrs Sarah Parsons, Harper Adams University College
Professor Nigel Peake, University of Cambridge
Dr Balazs Pinter, Aberystwyth University
Mrs Sue Pope, Qualifications and Curriculum Authority
Dr Atanas Popov, University of Nottingham
Mr Roger Porkess, Mathematics in Education and Industry
Dr Derek Raine, University of Leicester
Mr Matt Reed, Further Education National Consortium
Dr Mike Ries, University of Leeds,
Mr Paul Rispin, Peter Symonds College
Dr Carol Robinson, Loughborough University
Mrs Barbara Rundle, The Further Mathematics Network
Dr David Sands, University of Hull
Dr Tim Scott, University of Hull
Dr Stephen Siklos, University of Cambridge
Dr Bruce Sinclair, University of St Andrews
Ms Vivien Sloan, Specialist Schools & Academies Trust
Dr Alan Stevens, The Institute of Mathematics & its Applications
Dr Martin Symons, The Ogden Trust
Mr Peter Thomas, The Mathematical Association
Mr Charles Tracy, The Institute of Physics
Professor John Turner, University of Portsmouh
Mr Geoff Wake, University of Manchester
Professor Sir David Wallace, Isaac Newton Institute for Mathematical Sciences
Professor Graham Wilks, Keele University
Dr Clare Woodward, Durham University

Steering Group

Professor Mike Savage University of Leeds (Chair)
Mr Tom Button, The Further Mathematics Network
Mr Shaun Canon, The Further Education National Consortium
Professor Tony Croft, Loughborough University
Mrs Sue de Pomerai, The Further Mathematics Network
Ms Janice Gardner, EBS Trust
Mr Michael Grove, HEA MSOR Subject Centre
Dr Trevor Hawkes, University of Warwick
Professor Duncan Lawson, Coventry University
Dr Stephen Lee, The Further Mathematics Network
Mr Tom Roper, University of Leeds
Dr David Saunders, Symplekta Ltd
Mr Charlie Stripp, The Further Mathematics Network

Acknowledgements

The symposium and the publication of this report have been brought about by the commitment and enthusiasm of many people. For two years a steering group, chaired by Mike Savage and representative of schools, colleges and universities, has met to share information and ideas on how to address the issues relating to the teaching and learning of Newtonian mechanics discussed in this report. They formed the plan for the symposium.

Thanks now are due to those who have contributed to the symposium and ensured a widening of the debate and plans for action.

We are grateful to Sir Peter Williams not just for chairing the two days of presentation and discussion, but also for his participation in the planning. We are indebted, too, to Sir David Wallace, for his welcome to the Møller Centre and for his endorsement of what should be achieved. We should like to thank co-chairs, David Abrahams of the Institute of Mathematics and its Applications, John Morton of the Engineering and Technology Board, Andrew Ramsay of the Engineering Council, UK, and Tim Pedley of the University of Cambridge for leading our discussions. We are grateful too, to the presenters whose names are acknowledged within the report, and to every participant who contributed to productive planning for the future.

We should like to thank Michael Grove and colleagues from the Higher Education Academy Subject Centre for Mathematics, Statistics & Operational Research for their thoughtful and efficient organisation of the Symposium and for publication of the report; also Janice Gardner for her considerable help in compiling and editing the report.

Finally, special thanks are due to Andrew Blessley and the Clothworkers' Company for the generous funding which has made it possible to hold the symposium and produce the report to take this important issue forward.

Design and print production by em associates
Cover photograph by John Birdsall
Printed by Richmond Cavendish

Published by The Maths, Stats & OR Network

The Higher Education Academy Maths, Stats & OR Network

www.mathstore.ac.uk/mechanicsreport

ISBN 978-0-9555914-4-0