Review of Algebra Interactive

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I will begin by laying my prejudices on the table. My attitude to computer-assisted learning (CAL) is a mixture of enthusiasm for its potential and scepticism about the degree of success claimed for it so far. I am optimistic that in the longer term CAL will achieve great things but I have not seen many examples that exploit the educational and technical opportunities of the medium to the full.

Although I probably spend 90% of my working life interacting with a computer and largely enjoy it, I have not found the mouse and keyboard the most congenial way to learn mathematics. Give me a good book with a pencil and paper any day. Maybe I am too long in the tooth to adapt to new ways, but from talking to students, I get the impression this is not just a generation issue. I believe there are some inherent problems with computer-assisted learning for mathematicians:

- Writing mathematics on a computer screen, whether by drag-and-drop or keyboard entry, is still slow and clumsy. Few of us are fluent enough for spontaneous doodling with onscreen symbols
- Educational software for mathematics can be unreliable and unforgiving
- A clumsy, ugly, or dated user-interface can detract from the enjoyment of learning; one quickly takes for granted the slick and seductive graphics of expensive commercial software
- For physiological or aesthetic reasons, some people find it uncongenial to work at a computer

Thus the computer has some built-in barriers to overcome when promoting mathematical understanding.

On the positive side of the balance, the computer also has a lot going for it. Its potential for interactivity gives it a clear edge over a book.

- Hyperlinks and other navigational aids make it easier to present a mathematical narrative closer to the underlying logical structure
- Search facilities and conceptual linking offer fast and accurate cross-referencing to the intelligent learner
- Graphics, animated where appropriate, can provide striking visual insight into mathematical processes
- The power to calculate quickly and accurately, using the program's or the user's data, offers an unprecedented opportunity for mathematical experiment and exploration
- The computer provides a direct link to what is frequently an embarrassment of related information on the Internet
- As band-width and video-streaming improve, learning packages will be able offer a responsive human interface closer to the experience of a tutorial

And of course, for those who enjoy learning mathematics in the company of others, the computer is an ever-improving networking tool.

The arrival of the stylish-looking book *Algebra Interactive* on my desk set my optimistic juices running. Neatly embedded in its hard front cover was the CD-ROM, over-printed in familiar Springer colours of yellow and dark blue to dovetail with the cover design. "New!" proclaimed the little red sticker holding it in place. 'LEARNING ALGEBRA IN AN EXCITING WAY' the logo promised. I was ready to be impressed.

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The Preface explains that *Algebra Interactive* "differs in content from classical material on algebra in that it focuses on algorithms (constructive aspects) and real life applications; the presentation deviates strongly from classical algebra books in that it brings to life the basic concepts of algebra through interactive examples and visualizations". New Technology, it suggests, enriches the material with:

- many examples for experimenting
- dynamic illustrations throughout the text
- buttons focusing on specific aspects (proofs, examples, exercises, illustrations)
- on-line multiple choice tests
- on-line calculators pertaining to the subject covered
- various ways of cross referencing

In the review that follows I shall attempt to assess how well the book and CD measure up to these objectives.

The first thing to realise is that the CD is at the heart of the learning package; the book is seen as a junior partner. As the Preface explains: "Also, for your convenience, we provide this reader, a paper excerpt of *Algebra Interactive*. It contains most of the text from the main window (in standard font size) and a fair portion of the proofs, examples, remarks, applications and explanations from the secondary window (in a smaller font size)."

But I began with the 160 page book anyway, because I was at home at the time, *Algebra Interactive* (henceforth *AI)* does not run on MacOS, and my children were hogging the family Wintel machine. The book has eight chapters and an appendix on using the CD. The chapter titles give a good idea of the material covered: 1. Arithmetic; 2. Modular Arithmetic; 3. Polynomials; 4. Arithmetic modulo polynomials; 5. Permutations; 6. Monoids and Groups; 7. Rings and Fields; 8. Permutation groups. I rather hoped it might turn out to be a useful supplement to the algebra in our (the University of Warwick's) single-subject mathematics degree, but it awkwardly straddles modules in the first and second year, and makes no reference to quadratic forms and the canonical forms of matrices that are central to our second-year algebra syllabus.

The book is somewhat awkward to use by itself. It has no index and the results are labelled consecutively within each chapter; thus Chapter 4 begins with Definition 1, Theorem 2, Definition 3, Proposition 4, …, and ends with Fact 25. In Lemma 39 of Chapter 6, for instance, cosets are described by the usual equivalence relation, but the absence of a cross-reference made it hard to decide whether the concept of an equivalence relation had been introduced earlier or was 'assumed knowledge'. On the CD however, equivalence relations are succinctly covered in the 'prerequisites' section.

Some standard results are presented in their true colours as algorithms. For example, Euclid's algorithm (Algorithm 5 in Chapter 1) is stated as follows:

- *Input: two positive integers a and b*
- *Output: the* gcd *of a and b*

1. Replace (simultaneously)
   - *a by b and*
   - *b by the remainder of the division of a by b.*

2. Repeat Step 1 until b is equal to 0
3. Return a

A brief proof then justifies the form of the output. I felt that a student with programming experience would have a clear pedagogical advantage here. I counted a dozen similarly-formulated algorithms among the numbered results, which include around 73 designated 'theorems' and some 65 formal definitions. The book is elegantly type-set with generous margins. It has only 13 rather modest diagrams - the cogwheels in Figure 1 sadly lack cogs! - but the CD makes up for this visual short-fall.

I looked specifically for the real-life applications promised in the Preface. In Chapter 2 applications of modular arithmetic to public-key codes and radar detection are briefly introduced. Shift registers make an appearance at the end of Chapter 3 (as devices which "deliver a digit at every tick of a clock" like ghastly engines of torture!). Error-correcting codes, with their many modern uses, are discussed in the context of polynomial rings over Z_2 at the end of Chapter 4. Symmetry gets a fleeting mention in the Chapters 5 and 6 on permutations and groups, but the geometrical side takes a back seat. The long Chapter 7 on rings and fields concludes with a short description of Hadamard matrices and their applications to statistics and coding theory, but an opportunity is lost when the final Chapter 8 fails to mention any applications of group theory.

Now to the heart of the package, the CD-ROM, which runs on a PC with Windows 95/98/NT, LINUX, or UNIX, and requires 32 Mb RAM. It needs a recent version of Netscape (included on the CD) or MS Internet Explorer, and for LINUX and UNIX it requires the Java Runtime Environment, which can also be installed from the CD. I ran the CD on a 133 Mhz Pentium PC operating under Windows NT with 32 Mb RAM. I also attempted to set up the LINUX environment, but am not proficient at command-line computing and had to call for help. I interrupted our busy systems administrator with my elementary questions for a couple of hours, but not wanting to exhaust her goodwill, I did not complete the task.

When you insert the CD, your Web browser launches to display the *Algebra Interactive* splash screen in the browser window. Click on the animated logo and you get a graphical front page giving direct access to each of the eight chapters. A permanent frame, which stays on the left-hand side of the browser window, contains icons leading to the following features: Contents, Search, Tools (four specialized on-line calculators), Prerequisites, and Exercises. A 'kill gap' button allows you to shut down the GAP program when its calculations outrun your patience.

I found the navigation good but window management less so. The browser's main window contains the core material (the large-font contents of the book) with the definitions, algorithms, theorems, etc. (tableaux as they are called) set in boxes with a yellow background. At the bottom of this window, four arrows (left, right, up and down) let you move quickly between pages and levels; the alternative 'contents' icon in the left-hand frame takes you straight to the overview. Each tableau has a button with a drop-down menu offering you a variable choice of options such as proof, remark, exercise, gapplet, etc. as appropriate. Clicking a choice opens a second window, which is resizeable but hard to use on a 15" monitor without obscuring the main window. I switched between windows by opening and closing the second window; sometimes a third window is needed, for the on-line calculator for instance. The main screen also has a 'cool' button which launches a Java applet related to nearby material, again in the second window. I did not discover how to navigate within the second window, where, for example, a proof might appear sensibly containing hyperlinks to highlighted key words; having clicked on a key word, moved to the explanation and read it, I couldn't work out how return to the proof. The 'back' button on the main browser window had no effect on the second window.

Technical Issues

I found the software far from robust and not particularly transparent to use. In half a dozen sessions with the CD over a period of several weeks, the 'second-window' activities let me down in a variety of ways, sometimes as much as 50% of the time. Perhaps this was due to the limitations of the computer I was using (the sole Wintel machine in the Department), although its specifications were within the stated requirements. *Algebra Interactive* makes use of the computer algebra package GAP developed originally in Aachen and now at St Andrews; GAP is the engine for the four calculators (for handling modular arithmetic, polynomials, and permutations), for some of the exercises and tests, and for some 150 small programs called 'gapplets' designed to reinforce the algebra on the main pages. In addition, there are around 50 Java applets, which are graphically more sophisticated than the gapplets. I have 5 pages of hand-written notes of the technical difficulties I encountered and it would be tedious to list them. Here are a selection of the kinds of problem that arose:

- GAP error (e.g. dividing 7 into 999999) or number entered too large ($>10^{10}$)
- GAP screen garbled (frequently lacking the data-entry box) or output hard to decipher
- GAP takes too long (to calculate the HCF of x^5-1 and x^{10}-1, I let it run for 10 minutes before killing the program)
- Graphics run off the page (e.g. the Hasse diagrams)
- Problems with redrawing tables and pictures, and with scrolling
- Frequent 'error in the script' (with the whole set of Exercise 2.6, for instance)
- Failure of hyperlinks (in multiple-choice tests for example)
- Frequent blank grey screen when selecting a proof or a hint from a drop-down menu

Here are some examples of the lack of transparency that I also found frustrating:

- There are a number of interactive tables for concrete examples of binary operations (in monoids, groups, rings, etc). I could not make sense of the entries along the rows and columns and could not work out how to interact with them, in spite to the advice to drag colours. Clicking on the table cells highlighted the border but to no avail.
- I could not find the text for the prerequisites. The topics were highlighted hyperlinks, but clicking on them seemed to have no effect. (Much later, I discovered that text appeared in the bottom of the window off-screen and had to be scrolled to.)
- I could not work out which values to give the variables in the coding exercises.
- I could not make sense of the radar detection pattern graphics.

Some of these problems were due to my own obtuseness, but one should always aim low in such matters. Students vary greatly still in their ability to use computer graphical interfaces.

Educational Issues

Two questions spring to mind:

1 How can the package best be integrated into an undergraduate course?
2 How does the material engage the student and mediate the learning process?

The authors state that the material grew out of algebra lectures given at the Eindhoven University of Technology in The Netherlands, and I imagine they used the software as supplementary material to support the usual lectures and tutorials. They suggest that the material can be used for individual study, and also that the dynamic illustrations could be used by instructors to enliven their lectures.

The package contains a wealth of small tutorial programs, each designed to give insight and meaning to a theorem or an algorithm, exploiting the graphical and calculating power of the computer to enhance the enjoyment and understanding of the algebra. With clear guidance and supervision from a teacher, a student can use these valuable tools to experiment and to explore mathematical ideas. As these computer tutorials (the applets and gapplets) evolve, they will improve their educational effectiveness, and I hope the authors will welcome feedback from teachers using their material in other institutions. Having clear educational objectives in mind, they should not be afraid to analyse how each computer tutorial works to grab students' interest and to develop their knowledge and understanding. Classroom trials and student input should inform the package's steady development.

I do not believe that the package has yet reached the stage where it can be used as a free-standing module for independent study. If the authors have this as a longer term goal, more thought will need to be given to the motivation and explanation of the material. I had difficulty understanding some of the proofs, exercises, and computer tutorials, and I am sure a student of average ability would have similar problems. Preliminary mathematical experiments and problem solving help to prepare the way for the definitions and theorems, giving a sense of discovery and ownership of the material.

The technical shortcomings mentioned seem to corroborate my view that it takes huge resources to develop robust educational software to a professional standard, especially in a technical subject like mathematics. It is clear that the authors of this ambitious package have already committed much time and effort to the project. I hope that many university mathematics teachers will buy and use the package and pass on their experience with it in the classroom. In the absence of the big money available to commercial software firms, universities' best hopes for reliable software to enhance their teaching, learning, and assessment must lie in global cooperation, the educational equivalent of 'open code'. The authors hope, in the not-too-distant future, to produce a second version using the OpenMath standard for handling mathematical expressions. I wish them every success with their on-going venture.